begins simultaneously for any place of the earth, if $y > x$, then he should know that he lies on the east of the primary meridian because his Sun-rise happens to be earlier than the Sun-rise on the primary meridian. Also the difference $y - x$ gives the Des'āntara correction in time for his place. The converse is the case if he happens to lie on the western side of the primary meridian. The time at which the eclipse takes place on the primary meridian after the Sun-rise there which is obtained by computation is called Drik-grahaṇa - Kāla ; whereas the local time after Sun-rise observed by the observer is called pragrahaṇa-Kāla. Their difference is therefore the Des'āntara correction in time.

If the Des'āntara is to be got in yojanas, $\frac{T \times C}{60}$ is the answer, where $T = y - x$ and C is the rectified circumference of the earth, for, if a difference of 60 ghaṭis be there for C yojanas, what should be the distance in yojanas in order that the difference is T''? The answer is as given above.

Hence to obtain the positions of the Sun and the Moon at the beginning of the eclipse at the locality we have to add or subsract as the case may be $\frac{T \times \delta m}{60}$ where δm is the daily motion of the Sun or the Moon, and T is $y - x$ cited above, for, " If in 60 ghaṭis the motion be δm, what would it be in T ? " is the rule of three for which the answer is as stated above.

Now the question is when the week-day begins for the locality. It must be noted clearly, that in Hindu Astronomy the moment of Sun-rise at the primary meridian alone is to be reckoned as the beginning of the week-day universally. This convention is adopted for convenience. Thus the astronomical week day for any locality does not begin from the Sun-rise of the locality, but may begin earlier or later. This difference is given by $y - x$ cited above.

There is yet another subtlety in the commencement of the week-day, arising out of the latitude of the place. The former analysis pertains to the longitudinal difference. The difference arising out of latitude between the local Sun-rise and the Lanka-Sun-rise is given by what is called Chara-Kāla. Since the week-day begins at Lanka Sun-rise and the local Sun-rise differs from the Lanka Sun-rise not merely by a longitudinal difference but also by a latitudinal difference, to compute the actual beginning of the week-day before or after the local Sun-rise, we have to take into account both the differences cited above. In other words, computing the local Sun-rise and also the Lanka Sun-rise, we have to decide the beginning of the week-day before or after the local Sun-rise.

Verses 7, 8.   The correction called Bījakarma for the planetary positions.

The number of years from the beginning of the Kalpa divided by 12000, the remainder, or the difference of the divisor and the remainder whichever is less is to be divided by 200. The quotient in minutes of arc, multiplied by 3, 5, 5, 15, 2 respectively is a negative correction in the positions of the Sun, Moon, Jupiter, Venus, and the lunar apogee and multiplied by 1, 52, 2 and 4 gives the positive correction in the positions of Mars, Mercury, the lunar Node and the Saturn respectively.

Comm.   By the phrase ' The remainder or the difference of the remainder and the divisor ', it is plain that the corrections positive or negative increase for 6000 years and decrease for the next 6000 years. Bhāskara gives no reason for these corrections, but, we have to construe these corrections on the following rational grounds.   Bhāskara, however, says that the corrections were accepted by him on the basis of Āgama.   This Āgama —stipulation was there in Brahma–Sphuta–Siddhānta and was later incorporated by Srīpati also in his Siddhānta–S'ekhara and as such was

accepted by Bhāskara also. However, in Brahma Sphuta Siddhānta both as first published as an edition of M. M. Sudhākara Dwivedi and later by the late Rāmaswarupa S'arma in 1966, the verses 59, 60 of Madhyamādhikāra suggest that the corrections are negative in the case of all the planets; whereas both S'ripati and Bhāskara make them positive in the case of the latter four viz. Mars, Mercury, the lunar Node and Saturn. By this we have to construe that S'ripati and Bhāskara must have had before them a text which should have read 'ऋण' in the place of 'च' in the last pāda of verse 61. M. M. Sudhākara-Dwivedi did not notice this anomaly of the positiveness of the correction with respect to the latter four, but he remarked, however, that there was a prosodial lapse in the last pāda of verse 61, for which he offered a suggestion that instead of वेर्, we had better read वेरे:—This suggestion, no doubt, rectifies the prosody of the verse, but not the the anomaly cited above which was not noticed by M. M. Sudhākara Dwivedi. So, we have offered our own suggestion namely that in the place of च as mentioned above if we read ऋण, we not only rectify the prosodial error but also the anomaly referred to. Rāmaswarupa S'arma noted the anomaly but did neither refer to the prosodial error nor offer a correction. It seems that Rāmaswarupa S'arma did not verify the corrections stipulated from the verses 91, 92, 93 of Madhyamādhyāya of Siddhānta S'ekhara. In this latter work, there is another anomaly namely that in the case of Mercury, the number 62 is the multiplier and not 52. Makkibhatta, the ancient commentator had before him a text which read 62 in the place of 52, in all probability, a mistake of the scribe. M. M. Sudhākara Dwivedi is reported to have later pronounced that 52 must be the correct figure when this was brought to his notice as this number 52 was found both in Brahmagupta and Bhāskara. As reported by the editor of Siddhānta S'ekhara Pandit Babuaji Mishra, who mentions this latter pronouncement of Sudhākara Dwivedi his teacher, also says that Sudhākara Dwi-

vedi suggested the reading द्विशार in the place of द्विरस of
verse 93 of Siddhānta S'ekhara. The fact that Makkibhatta
commented द्विरसङ्गुणं as द्विराशिसङ्गुणं shows that he did
not consult Brahmasphuta Siddhānta in this place; also,
he must have had a manuscript before him which scribed
द्विरस in the place of द्विशार. Using श व, ब, indiscretely
is not uncommon in many books of North India, from a
long time and the scribe of the manuscript probably having
used ब in the place of श and then by an oversight a
latter scribe having inverted शर as रस, Makkibhatta must
have commented like that.

Incidentally a remark may be made here about Makki-
bhatta. He was evidently a keralite because he used letters
to signify numbers as was a common practice among the
Kerala Astronomers, and as he also commented upon Brihad-
Bhaskarīya. Further, it is interesting to note that he
wrote in his commentary under verse 39 of the Sauhana-
dhyāya of Siddhānta S'ekhara viz. " मभ्रमोन्ग तरमण्डलान्तरं
सावनानि कुदिनानि लानिवा ", " भू मे: प्राङ्मुखी भ्रमति " etc". This
idea shows that he accepted Aryabhata's verse " अनुळोम गति:
etc " implying that the earth is rotating.

Bhāskara says that the Bīja correction mentioned was
purely based on Āgama and Upalabdhi (meaning authority
and observation'). M. M. Sudhākara Dwivedi seems to
have reiterated the same as reported by Babuaji Mishra, in
a foot-note. Kamalākara, condemned this Bijakarma as
it was unwarranted and had no proof.

A rational explanation as to why this Bīja-Karma was
prescribed either by Brahmagupta himself or some autho-
rity which he seems to have accepted may be given as
follows. The small differences in the numbers of sidereal
revolutions or what is the same the minute differences in
the accepted daily motions of the planets and the assump-
tion of a conjunction of all the planets and planetary
points at the beginning of Kalpa, which is beyond proof,

resulted in a difference between the computed planetary positions and their observed positions. So, the originator of this Bija-Samskara, noting the differences in his own time devised a formula, which could account for those differences. But this formulation was bound to go wrong in later times as long as the daily motions are not corrected to the minutest extent possible and as long as the fundamental basis of the conjunction of all the planets and planetary points is not proved. This seems to be the reason why so may texts were written incorporating small differences in different times as reported by Ganeṣa (1507 A.D.) in his work Brihat-Tithi-Chintāmaṇi in the words " The calculations of planetary positions according to the methods indicated by Brahma, Vaṣishtha and Kasyapa Siddhantas held good in their own times, but grew obsolete later; Then Maya, the demon at the end of Krita obtained the science from the Sun God, which again grew obsolete in this Kaliyuga wherein parāṣara began to hold the ground for a good length of time. Then Āryabhata rectified the methods; when even those methods grew obsolete, Durga-Simha, Varāha Mihira and others set them right. Again Brahmagupta came into the picture to rectify the methods by his own observations. Then Came Keṣava (Gañeṣa's father) who rectified further. After a lapse of sixty years, his son Ganeṣa has now to correct the Science. If this also grows obsolete (as it is bound to) in course of time, let others again rectify it by observing conjunctions of the Moon and planets with the asterisms."

Obsoleteness arises out of two contexts, one a justifiable situation and the other based upon a wrong premise. The first is as follows. Suppose as a first approximation we take the length of an year as 365 days. We will have committed an error nearly $\frac{1}{4}$ of a day, so that the error accures to a day in 4 years. Thus the convention of the leap year arose so that during four years we give a day more to February. Here again we have overestimated the error by nearly $\frac{1}{100}$th of a day. Hence in 400 years the

above correction leads to an error of a day. So, it is that we pronounced that out of the years 20C0, 2100, 2200, 2300 A.D., the year 2000 A.D. alone is a leap year, and not the remaining, the convention being that the number of the century, here 20, must be also a multiple of four. On this back-ground, suppose we prepare a manual called a Karaṇa grantha taking the length of the year to be 365·25 days. It works alright for some time but in the course of 400 years the error will have reached to as much as one day. Thus a manual like the above works only for a short time and the approximation made gradually brings in a divergence on account of which such a manual grows obsolete. That is why one Narasimha who happened to prepare a manual in 1333 Saka year (1411AD) opens his work with the words " तिथिचक्रं यत्प्रणीतं मल्लिका-र्जुनसूरिणा, कालेन महता तस्मिन् खिलीभूते तदादरात्, नौपुरीसिङ्गयार्यस्य नरसिंहसुनुना एतदेव स्फुटतरं क्रियते सौरसम्मतम् " i.e. "In as much as a manual named Tithi-Cakra prepared by one Mallikār-juna Sūri long ago, based upon the Sūryasiddhānta has now diverged far from the Sūryasiddhānta (on account of the approximations made transcending the limits of negligibi-lity) I, the son of one Singaya belonging to a place named Nau-puri (probably Vada-palle of the East Godavary Dt.) am rectifying it and bringing it to accord with the Sūrya Siddhānta again."

This kind of obsoleteness arising out of inevitable ap-proximations that have to be made in the preparation of manuals is permissible. But Suppose the premise of the manuals itself is incorrect, then the rectification of the manuals is no good so long as the data given in the pre-mise are not corrected. There are two fundamental defects in the ancient works according to a modern analysis namely (1) The Supposition that all the planets were in conjunction at the Zero-point of the Zodiac in the begin-ning of a Mahāyuga (2) Small variations in the constants like the daily motion of the planets and the like. Accord-ing to the modern interpreters of Hindu Astronomy the

first premise was not correct. According to them, some astronomers having observed the daily motions of the planets or what is the same the sidereal periods of the planets to a sufficiently good approximation calculated back or extrapolated a date on which these planets should have been in conjunction at the Zero-point of the Zodiac. The extra-polated date was naturally wrong to some extent because the sidereal periods found could not but be correct only to a particular degree of approximation. Thus a little alteration in the number of sidereal revolutions alone or the number of days in a Mahayuga made to suit the observed positions at a particular epoch would be only a tinkering of the problem and not a true solution. Thus Hindu Astronomy could be saved and its methods could still be followed provided instead of trying to presume a date at which all the planets were in conjunction (No doubt in the long bosom of time, such a presumption also could not be ruled out) correctly observed positions of the planets by the help of modern instruments were taken as the basis of an epoch and thereafter using more correct values of the constants such as the sidereal revolutions, maximum equations of centre and maximum S'ighraphala, obliquity of the ecliptic etc. The second defect cited above thus being removed, and the original premise being changed, the methods of calculation still hold good and there would be no necessity to be going on with tinkerings of the problem.

The Bija-correction which we are commenting upon was rightly criticised by Kamalākara as irrational though he himself fanatically tried to uphold Surya Siddhānta. Even today there are a good number of the traditional Hindu Astronomers who do hold that the Sūrya Siddhānta was revealed to Maya at the end of Kritayuga in spite of the fact that scholars like M. M. Sudhākara Dwivedi pronounced that it was an extra-polated work shortly after the time of Brahma-Guptāchārya. It is interesting to note that Bhāskara, a very rational astronomer, had before him the verse " त्रिघनकृत्वो युगे भानां चक्रं प्राक् परिवर्तते " of the

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Sūrya Siddhānta (verse 9 ch. 3). He did not give it the interpretation that was later put upon it through the two subsequent lines "तद्दो चित्रा" etc., which lines were not there evidently in Bhāskara's time. Without these two latter lines the rate of precession was too small to be accepted by Bhāskara and so he chose to follow Munjāla rather than the Sūrya Siddhānta. Our Traditional astronomers today have no reservation to accept the greatness of Bhāskara and worship him though they do not question what necessity Bhāskara had to write another treatise and that too basing it upon the Āgama accepted by Brahmagupta and not Sūrya Siddhānta, when there existed Sūrya Siddhānta before him and from which he had no objection to quote verses like "अदृश्य पाः कालस्य मूर्ियः" etc. (verse I ch. 2.)

The Bīja-correction first incorporated by Brahmagupta and later followed by a good number of astronomers because S'ripati and Bhāskara accepted it, will not be acceptable to modern astronomers, though it might have worked well at the time of Brahmagupta and for some years later. The reason is that it is construed only as a tinkering of the defect as explained before.

It is also to be noted that the originator of this Bīja-correction did not make it secular i.e. valid for all time increasing without a limit, for, then, the respective corrections transcend all limits and render the corrections meaningless. So, he said that the corrections would be increasing for 6000 years and thereafter begin to decrease to nothing. They were Zero at the beginning of the Kali because all the yugas are multiples of 12000 years. Also the maximum correction is in the case of Mercury $\frac{6000}{200} \times 52 = 1560' = 26°$. Let us see how far this is justifiable. The daily mean motion of Mercury as given by Bhāskara is $4° - 5' - 32'' - 18''' - 28''''$ whereas as per modern astronomy it is $4° - 5' - 37\frac{7}{8}''$ approxly. So there is a positive error of $5''\frac{23}{40}$ which will accrue to $18' - 35''$ in

course of 200 years. But as per the Bīja-correction it should be 52'. Hence it is a fact that there is a positive error but not so much as indicated. But it must be noted that Mercury's orbit has the highest eccentricity of as much as .2, and the observer who stipulated the correction must have observed when Mercury was near its perihelion, where the error could have been as much as indicated and even more. Similarly on close analysis it could be proved that the Bīja-correction should have been as indicated, say, roughly about 3300 Kali era, which might be roughly the date of its stipulation.

Verses 9, 10. Concluding verses of the Madhyādhikāra.

If the work is made more voluminous by describing various methods which are easy and interesting to un-intelligent people, learned men look down upon such a work as indulging in unnecessary verbosity. Hence the volume of a work does not add to its greatness; So I have made my work neither voluminous nor brief-worded. The reason is that both the intelligent as well as the unintelligent people are to be enlightened.

For the sake of clarity of exposition, different ingeni-ous methods being used in such a way that the work does not exceed the normal limits of the previous works, and in corporating as far as possible unit numerators, fractions having numerators and denominators mutually prime, using methods of interpolation and reduction, making use of different kinds of denominators and numerators in many ways, this kind of treatment must be given to a work of this nature by an intelligent man.

Comm. Easy.

Before we proceed to the next chapter, we shall add here tables of astronomical constants as given by different authorities, which will help comparison and appreciation of the work.

TABLE 1

Number of sidereal revolutions of planets and planetary points in a Kalpa

	Modern Sūrya Siddhānta	Bhāskara	Āryabhata	Khaṇḍa Khādyaka	Mahā-siddhānta
Sun	4320000000	4320000000	4320000000	4320000000	4320000000
Moon	57753336000	57753300000	57753336000	57753336999	57753334000
Mars	2296832000	2296828522	2296824000	2296824000	2296831000
Mercury	17937060000	17936998984	1793702000	17937000000	1793705467l
Jupiter	364220000	364226455	364224000	364220000	364219682
Venus	7022376000	7022389492	7022338000	7022388000	7022371432
Saturn	146568000	146567298	146564000	146564000	146569000
Moon's Apogee	498203000	488205858	488219000	4982l9000	488208674
Moon's Node	232238000	232311168	232226000	232226000	232313354

TABLE 2

	According to Sūrya Siddhānta	According to Bhāskara
(a) Number of mean solar days in a Kalpa	1577917728000	1577916450000
(b) Number of Adhika-māsas in a Kalpa	1593336000	1593300000
(c) Number of Kshayā-has in a Kalpa	25082252000	25082550000
(d) Diurnal revolutions of stars	1582237828000	1582236450000
(e) Tithis in a Kalpa	1603000080000	1602999000000

TABLE 3

Daily mean motions of planets

	Brahmagupta Sripati and Bhāskara
Sun	0-59'-8"-10"'-21""
Moon	13°-10-34-53-0
Mars	0-31-26-28-7
Mercury	4°5'-32"-18-28
Jupiter	0-4-59-9-9
Venus	1-36-7-44-35
Saturn	0-2-0-22-51
Moon's Apogee	0-6-40-53-56
Moon's Node	0-3-10-48-20

SPAṢṬĀDHIKĀRA — RECTIFICATION
OF PLANETS

Introduction. In the Bhagaṇādhyāya section of the previous chapter Bhāskara gave under the Caption Bhagaṇopapatti his proofs as to how the ancient is might have obtained the number of sidereal revolutions of the planets and the planetary points called apogees or aphelia and Nodes. But in trying to give those proofs, he was aware and he confessed also in so many words that some of his proofs at least were obsessed by what is called Itarētarāśraya-Doṣa i.e. "answer begging the question" It is worth hearing his words in his commentary under verses 1-6 of the section cited above —" That the planets, and the planetary points perform so many revolutions in a Kalpa, is essentially conveyed by the Āgama i.e. the S'āstra (which is to be taken on faith). That Āgama, got diversified i.e. there are many versions of that Saience, due to the defects of scribes, the teachers and the students and due to a long lapse of time from the originators of the Āgama. That being so, the question arises as to which of the versions is to be trusted as the right authority. If it be said so, in mithematics only an āgama which could be proved also should be taken as authority. Such a number of revolutions as are obtained by proof, is to be accepted. Even that could not be (a proof) ; for, a great scholar could just understand the proof and by that proof alone, it is not possible to know the exact number of revolutions (in a kalpa), for, a man's longevity is not much. In the proof that could possibly be given, the planet's position is to be observed and noted every day, during the entire course of its revolution. Thus Saturn Completes its sidereal revolution in about 3) years. The apogee of the Sun and the aphelia of the planets have their revolutions running into hundreds of years. Hence the observation of one complete revolution (of such a planetary point) is beyond the capa-

city of a mortal. Hence great astronomers accept such an āgāma as would give results which accord with observations during their times, and such a one as was formerly accepted by a very intelligent astronomer. Then they produce their own works exhibiting their own Skill in the Science and refuting wrong notions of others. Their idea is ' Let the Āgama we take as an authority be whatever it would be Let us show our own skill in the course of our work '; just as in this work, the āgama accepted by Brahmagupta is taken on faith as the authority. Then it might be argued " Better not attempt at trying to prove how the numbers of sidereal revolutions were arrived at. Even if a proof be attempted, that proof *would be obsessed by the* ' *Itarētara-s'rayadōṣa* ' (cited above). Nevertheless we shall give a brief proof, That ' itarētarās'rayadōṣa ' is apparently a dōṣa i.e. an apparent defect; for, different proofs could not be adduced simultaneously. The proof will now be given".

These words indicate that even such a highly rational and supremely intelligent astronomer like Bhāskara could not set aside his faith in our āgama and attempt at a pure and rigorous proof, which would not invoke the āgama-Let us see where in his proof he does commit the so-called itarētarās'rayadōṣa and where he invokes the āgama. Also we shall try to construct a proof, of course to a good extent on the lines on which Bhāskara tries to give his proof, but at the same we shall not invoke the āgama. where he does, but try to proceed purely on a rational basis. We shall take up the proof under verse 18, in its appropriate context. We shall now proceed with the text upto that point, which gives a brief sketch of the Hindu trigonometry.

Verse 1. In as much as true positions of the planets alone are required to decide auspicious moment; for journeys, marriages, celebrations pertaining to temples, astrology and the like, we shall now give the methods of rectifying the mean positions of the planets so as to accord with their observed positions.

Comm. Clear.

Verse 2-9. Obtaining the sines of the angles and tabulation of the sines.

The planet deflected to the true position from the mean lies at the end of a half-chord (which is the Hindu sine of an angle) so that many processes pertaining to a planet are carried through Sines of angles; hence the word half-chord alone is connoted in this work wherever the word Jyā meaning a chord is used.

The lengths of these half-chords (or the Hindu Sines) for angles increasing from 0° to 90° at intervals of $3\frac{3}{4}$° are as follows—225', 449', 671, 890, 1105, 1315, 1520, 1719, 1910, 2093, 2267, 2431, 2585, 2728, 2859, 2977, 3084, 3177, 3256, 3321, 3372, 3409, 3431, 3438. The ut-kramajyāīs or the Hindu versed-sines are respectively 7, 29, 66, 117, 182, 261, 354, 461, 579, 710, 853, 1007, 1171, 1345, 1528, 1719, 1918, 2123, 2333, 2548, 2767, 2989, 3213, 3438.

The word Tribhajyā or Trijyā is half-diameter. The word jyā khandas used by pandits connote the differences between successive sines.

Comm. There is a difference between modern trigonometrical sines and the Hindu sines as detailed below (Ref.
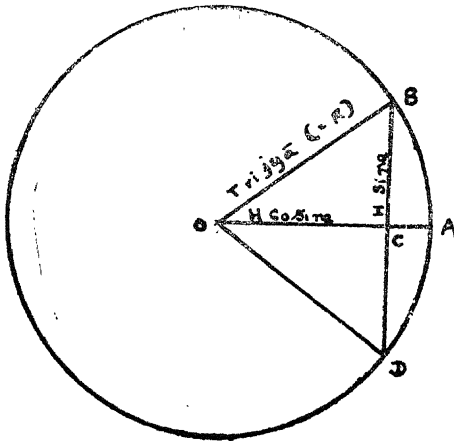
Fig. 6

fig. 6 overleaf). Let (0) be a circle i.e. a circle with centre
'0'. Let AB be an arc called 'Chāpa'; let BC be drawn
perpendicular on OA; then BC is half of the full chord
BD (known as jyā). The half-chord Ardha-jyā is itself
spoken of as jyā for convenience and is the Hindu-sine of
the arc or chāpa AB. In Hindu trigonometry 'angle' is
connoted by the arc corresponding to it and as such spoken
of as chāpa. OC is spoken of as the Hindu-cosine or Koti-
•jyā and CA is called the ut-kramajya or the Versed-sine.
The radius O B is called trijyā and let us connote it by R.
To differentiate between the modern terms and the Hindu
terms, we use the words H. Sine, H. Cosine, H. vers-sine
for the Hindu sine, the Hindu cosine and the Hindu vers-
sine respectively. Also the radius R is generally taken to
be 3438' which, we know to be the approximately the
minutes in a radian. To talk of a length in minutes ap-
pears rather odd but no confusion need be there, for, an arc
of length R subtends 3438' at the centre. It is called Trijyā
for the reason that it is the H. sine of 3 Rasis or 90°.
A Rasi is equal to 30° because the ecliptic circle of 360°
is divided into 12 Rasis Mesha, Vrishabha etc. meaning

Aries, Taurus etc. The names of the Rasis in Sanskrit and the modern English words we use for them have the same meaning, which raised a suspicion in the minds of many orientalists that the Hindu Astronomy drew upon the Greek. Many scholars of India assert that the Greeks derived this knowledge from the ancient Hindus; but we shall not enter into the controversy here. It may be noted also that the Sanskrit names of week-days have the same meaning as Sunday, Monday etc.

On the basis of taking trijyā equal to 3438', the other H. sines or half-chords are also expressed in minutes. Generally twenty-four H. Sines are given in a quadrant and to obtain the H. sine of an angle intermediate, a formula for interpolation also is given. Also the method of calculating the H. sines for every degree is given, as we shall see shortly. In the table of 24 H. sines, the first is H. sine 3°-45' or H. sine 225' and this is approximately taken as 225' because in fig. 6 if $A\widehat{O}B = 3°-45'$, the H. sine BC will be almost equal to the arc AB. The H. Vers-sines are also given to get the corresponding H. Cosines easily, for, H. Vers sine 3°-45 = R — H. Cos 3°-45 = 7' means H. Cosine 3°-45' = 3431 = H. sine $(90°-3\frac{3}{4}°) = H_{23}$ where we use the notation H r to mean the rth H. Sine.

Now we propose to give here some essential formulae used by the Hindu astronomers, as given in the golādhyāya by Bhāskara under the caption jyotpatti-krama. Incidentally it may be noted that H. sine $\theta = R$ sine θ where sine θ is the modern sine of the angle $\theta°$. Similarly H. Cos $\theta = R$ Cos θ and H vers $\theta = R$ vers θ. Thus when we have an equation of the type. Sin δ = Sin ϕ Cos z + Cos ϕ sin z sin a in modern astronomy arising out of the famous spherical triangle PZS where P is the Celestial Pole, Z the Zenith and S the position of the Sun or a Star, the Corresponding Hindu formula would be R^2 H Sin δ = RH Sin ϕ H cos Z + H cos ϕ H sin Z H sin a. Occasionally the

radius is taken to be 120, and the corresponding H sines are called Laghu-jyās or simpler H sines used where great accuracy is not required. Srīpati took the radius to be 3270 units in addition to 120 as did Brahmagupta. Munjāla took 488' and some others some other values also. Out of these 3438' alone has a right significance (Vatēswara took 3272)

Bhāskara says under verses 1 to 5 under Jyotpatti-Vāsanā in the Golādhyāya that the Hindu astronomers got the values of the main H sines of 30°, 45°, 60°, 18° and 36° by inscribing regular polygons in a circle. They are called the pancha-jyakās or the fundamental H sines. From these the others were calculated according to the methods given by Bhaskara as follows.

To start with, we have the fundamental formula

$$H \sin^2\theta + H \cos^2\theta = R^2 \text{ (from fig-6)} \qquad I$$

In addition to this formula, Bhāskara gives another formula (verse 10, 11 Ibid) $H \sin \theta/2$

$$\overline{\theta + H \text{ vers}^2 \theta} =$$

In the commentary under the above verses, he has given the method by which II was obtained (Ref. fig. 7) $BM = H \sin \theta$ where $\widehat{AOB} = \theta$; also $AM = H$ vers θ and $AB^2 = AM^2 + MB^2$. Let N be the mid-point of AB.

$$= AN = H \sin \theta/2$$

$$H \sin \theta/2 = \tfrac{1}{2}AB = \tfrac{1}{2} \sqrt{AM^2 + M}$$

$$= \tfrac{1}{2} \sqrt{H \sin^2 \theta + H \text{ vers}^2 \theta} \text{ which}$$

proves the first part of II. Again from the right—angled triangle ABC, $AB^2 = AM \cdot AC = H$ vers $\theta \times 2R$

$$= \tfrac{1}{2} \sqrt{2R \; H \text{ vers } \theta} = \sqrt{}$$

which proves the second part.
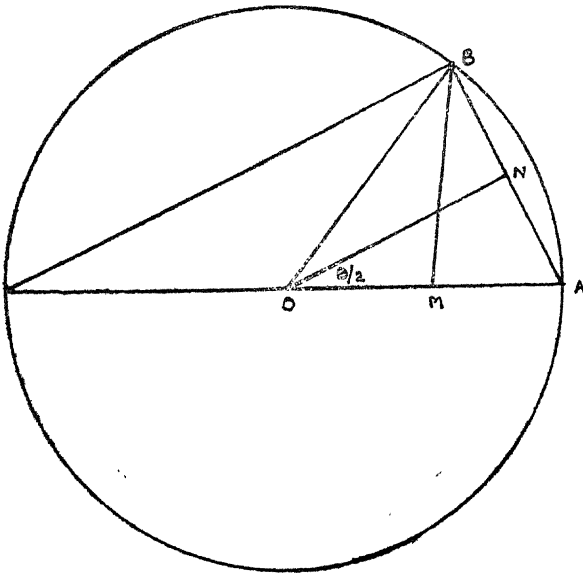
Fig. 7

In the Commentary under verses 1—25 ibid, Bhāskara tells us how formulae I and II are used to construct the table of 24 H sines. To start with, the four H sines of 30°, 45°, 60° and 90° which may be denoted by the symbol Hr where $r=8$, 12, 16 and 24, are known. Now using the formula II, H$_4$ is obtained from H$_8$, H$_2$ from H$_4$ and H from H$_2$. Similarly from H$_{12}$, H$_6$ and H$_3$ are successively obtained. Now using formula I, H$_{20}$, H$_{22}$. H$_{23}$, H$_{18}$. H$_{21}$ are obtained respectively from H$_4$, H$_2$, H$_1$, H$_6$ and H$_3$. Now again from H$_{20}$, H$_{22}$, H$_{18}$, we obtain using formula II H$_{10}$, and H$_5$, H$_{11}$, H$_9$ respectively. Formula I gives again H$_{14}$, H$_{19}$, H$_{13}$, H$_{15}$ from the above. H$_{14}$ gives H$_7$ and H$_7$ gives H$_{17}$ using formula II and I respectively. Thus the table is Completed.

Then Bhāskara poses the problem as to how a table of the H sines could be computed when a quadrant is divided into 30 equal parts. He says that formula I and II do not suffice in this behalf and shows how they do not, as follows

in the same commentary cited above. To start with, the H sines of 18°, 30°, 36°, 45°, 54°, 60° are known. They are respectively H_6, H_{10}, H_{12}, H_{15}, H_{18}, H_{20}. Also H_{30} i.e. H sin 90° = R is also known. Formulae I and II will help us to derive,

from H_6, H_3 and from H_3, H_{27}; also from H_6, we derive H

From H_{10}, H_5 and from H_5, H_{25} and again

from H_{18}, H_9 and from H_9, H_{21} are derived.

The remaining H sines sixteen in number cannot be got from either of the formulae.

To meet the situation Bhāskara gives other formulae of his own discovery as he says "प्रवक्ष्येऽथ विशिष्टमस्मात्" i.e. "I shall tell something more than this. These formulae he gives in the verses 12 to 15. They are

$$\text{Sin} \left(\frac{90 \pm x}{} \right) = \sqrt{R^2 + R\,H\,\text{Sin}\,x}$$

$$\overline{\text{sin}\,x + H\,\text{sin}\,y)^2 + (H\cos x - H\cos y)^2} \quad \text{IV}$$

$$\sqrt{\frac{(H\,\text{Cos}\,x - H\,\text{Sin}\,x)^2}{2}} = H\,\text{Sin}\,(45 - x) \quad \text{V}$$

$$R - \frac{2\,H\,\text{Sin}^2\,x}{R} = H\,\text{Sin}\,(90 - 2x) \quad \text{VI}$$

These formulae correspond to the modern formulae

$$\text{Sin}\left(\frac{x-y}{2}\right) = \sqrt{\frac{(\text{Sin}\,x + \text{Sin}\,y)^2 + (\text{Cos}\,x - \text{Cos}\,y)^2}{2}}$$

$$\frac{(\text{Cos}\,x - \text{Sin}\,x)^2}{2} = \text{Sin}\,(45 - x)$$

$$1 - 2\,\text{Sin}^2\,x = \text{Cos}\,2x \quad \text{respectively}$$

These formulae imply a knowledge of the expansion of Sin $(x \pm y)$ which is given in verses 21, 22 in the form

$$H \sin (x \pm y) = \frac{H \sin x \, H \cos y \pm H \cos x \, H \sin y}{R}$$

The formula $H \cos (x \pm y)$ is got from VII by putting $90 - (x \pm y)$ for $x \pm y$.

To construct the remaining sixteen H Sines Bhaskara directs us to use his formula IV wherein taking $x = 27°$, and $y = 15°$, we have H_2 which gives H_{28}. From H_{28} we have H_{14}, H_7 and H_1 from H_2. Then H_{16}, H_{23} and H_{29} are got from H_{14}, H_7, and H_1 respectively. From H_{16} again we have H_8, and H_4 which in turn give give H_{22} and H_{26}. H_{26} gives H_{13} which in turn gives H_{17}. · H_{22} similarly gives H_{11} which in turn gives H_{19}. Thus the table is complete.

Verses 16 *to* 20. (*Ibid.*) give us the method of constructing the table of 90 H sines in a quadrant, through the formula $H \sin (x \pm 1)° =$

This formula is got evidently by interpolating from his knowledge of $H \sin 3°$ and $H \sin 3\frac{3}{4}°$. He also gives that $H \sin 5\frac{3}{4}°$ is more correctly equal to 224', 51". In the table of 24 H Sines, he gives the formula $H_{r+1} = Hr \left(1 - \frac{1}{497}\right) + H \cos x_r \times \frac{100}{1529}$ which could be similarly got by interpolation.

The determination of $H \sin (A+B)$ from $H \sin A$, $H \sin B$, $H \cos A$, $H \cos B$ is called Samāsa-Bhāvanā and that of $H \sin (A-B)$ is called Āntara-Bhāvanā, whereas computation of $H \sin 2A$ from $H \sin A$ and $H \cos A$ is called Tulyabhāvanā. The word Vajrābhyāsa is used for 'Cross-multiplication' in this context.

We shall now prove how the formula for $H \sin (x+1)°$ led Bhāskara to arrive at the differential formula

δ (Sin x) = Cos x δx.   Since H Sin $(x+1)° =$ H Sin x + $\dfrac{\text{H Cos x} \times \text{H Sin 1°}}{R}$ approximately,   H Sin $(x+1)° -$

H Sin x° = H Cos x $\times \dfrac{\text{H Sin 1°}}{R} =$ H Cos x $\times$ a constant.

Hence Bhāskara could see that the variation in the function H Sin x is proportional to H Cos x. Let it be now required to find the increment in H Sin x for an increment δx in x where δx < 60'.   Let H Sin $(x+1)° -$ H Sin x = $\dfrac{60' \times \text{H Cos x}}{R} =$ y where y is called the Bhogya-Khanda.

Then Bhāskara argues "If for an increment of 60', there is an increment of y, what shall we have for δx ?".

The answer is $\dfrac{y\delta x}{60} = \dfrac{60 \times \text{H Cos x}}{R} \times \dfrac{\delta x}{60} = \dfrac{\text{H Cos x } \delta x}{R}.$

Hence H Sin $(x + \delta x) -$ H Sin x = δ (H Sin x) =

H Cos x $\times \dfrac{\delta x}{R}$ which corresponds to δ (Sin x) = Cos x δx.

Bhāskara is thus the first mathematician to have perceived this differential formula 500 years before Newton and Leibnitz.

In the context of the preparation of the table of 24 H Sines which was there in Āryabhatiya as well as Sūrya-siddhānta, we have to offer the following remarks. The method of the construction of this table was the subject-matter of some study by S. N. Naraharayya and A. A. Krishnaswami Ayyangar[1]. In this study it was supposed that the method was based on finite differences according to the verses of the Sūryasiddhānta 15 and 16 of chapter II. The articles referred to reveal the difficulty in constructing the sine-table following this method. Some subsidiary corrections in the verse "वक्रविशाऋ विशाऋ

1. S. N. Naraharayya—Journal of Indian Math. Society, Vol. XI—First Series Pages 105-113.

2. A. A. Krishnaswami Ayyangar—J. I. M. S., Vol. XV—First Series Pages 121-126.

quoted
from the Brahma Siddhānta by Ranganātha in his com-
mentary of Sūryasiddhānta were alluded to in the articles
cited but no satisfactory mathematical explanations were
given by them. We shall give hereunder a satisfactory
explanation of the matter discussed in the articles.

In the first place it may be noted that in the table of
those 24 H sines, the sixteenth as given by Bhāskara
namely 2977 is more correct than that given in the Sūrya-
siddhānta namely 2978 (Lakshmi Venkateswara press edi-
tion 1955 Bombay).

In the course of the Commentary under the verses 15,
16 of the Sūryasiddhānta, Ranganātha gives the hint which
must have been at the back of the mind of the author of
the Sūryasidhhānta when he gave the rule to construct the
table cited. Just as δ (H Sin θ) = $\dfrac{H \cos \theta \delta \theta}{R}$, Similarly

the formula δ (H Cos θ) = $\dfrac{- H \sin \theta \, \delta \, \theta}{R}$ must have

been known to the author. The negative sign means that
the successive differences of the H sines namely 225, 224,
222, 219 etc. are decreasing and also that the successive
differences of these differences are increasing according to
the H sine. Just as Bhāskara could see that the H sines
were increasing and the successive differences of the
H sines were in Kotijyānupāta i.e. in direct ratio to the
H Cosine at their respective place, similarly, the author of
the Sūryasiddhānta could see that the second differences
cited above were in Kramajyānupāta as hinted by Ranga-
nātha.

From the formula $\delta \left(\dfrac{H \cos \theta \, \delta \, \theta}{R} \right) = \dfrac{- H \sin \theta \, \delta \, \theta^2}{R^2}$

putting θ = 90°, we have the second difference numerically

$$\frac{\times}{R} \quad = 14' - 43'' - 30''$$

Ranganātha made a mistake in taking this to be

$$= 15' - 16'' - 48''' - \text{Even } \frac{\delta\theta^2}{R} \text{ is approximate}$$

and a more correct value of the second difference would be 14' — 47" approximately. Ranganātha then argues that taking this second difference to be 15 for the H sine 3438 ' what will it be for the H sine 225' ? ' The answer would

be $\frac{15 \times 225}{3438} = \frac{15 \times 25}{382} = \frac{375}{382} = 1'$

So, the second difference in the beginning of the table

happens to be 1' ie $\frac{225}{225}$. This led the author of the Surya

siddhānta to use the words " तद्विकलत्वोन ". This being an approximate formulation, naturally necessitated a second formulation where the approximation led to an error of 1' through the verse "एकविंशाच्च विंशाच्च etc." This second formulation intended to make a correction, was done in the wake of a correct calculation through the formulae I & II which were known even prior to Bhaskara.

Verses 10, 11. To find the H sine of an intermediate angle. Suppose it is required to find the H sine of an •angle $\theta°$ ie $\theta \times 60'$. Divide this by 225; the quotient gives the previous H sine. Then $\frac{R \times D}{225}$ where R is the remainder, and D the difference between the previous and next H sines, added to the previous H sine gives the H sine required.

Comm. The formula is evidently based on an application of rule of three.

Verse 11. To find the angle when the H sine is given Suppose the H sine of an angle is given to be x'. Subtract the greatest H sine that could be subtracted from this.

15

Suppose the H sine of $\theta°$ could be subtracted. Let the remainder be r. Then $\dfrac{r \times 225}{D}$ where D is the difference between the previous and next H sines, added to θ gives the angle corresponding to x'.

Comm. Evidently this is the converse of the previous process and this also is based on Rule of three.'

Verses 12—15. The H sine of the obliquity of the ecliptic taken to be 24° is 1397. Now, the successive differences of the H sines will be given (on the basis of taking R = 120) which are known as Laghu-Jyās intended for ease in Computations, namely 21, 20, 19, 17, 15, 12, 9, 5, 2. These are given for intervals of 10°, so that if it be required to find the H sine of x°, let q be the quotient and r the remainder when x is divided by 10. q gvies the number of the previous H sine. Then $\dfrac{r \times D}{10}$ where D is the next difference or jyākhanda as it is called, added to the previous H sine gives the required H sine. In this table the H sine of 24° is 48' — 45". Also the H versines in this table are got by the reverse differences. To get the angle $\theta°$ for a given H sine say x' subtract the sum of as many differences (Jyā—Khandas) as could be from x. Let the remainder be r. Then $\dfrac{r \times 10}{D}$ where D is the next jyā—Khanda added to the previous angle upto which the jyākhandas have been subtracted, gives the required angle. The H sine will be more accurate if the Bhōgya-Khanda or the next H sine—difference is rectified (as per the rule of interpolation next given).

Comm. H vers θ = R—H Cos θ = R—H sin $(90-\theta)$ so that H verse $3\frac{3}{4}°$ = 3438—H sin $(86\frac{1}{4}°)$ = 3438—3431 = 7 as given in the previons table. Similarly in the above table of Laghu—Jyās, H vers 10° = R — H cos 10° = 120 — H sin 80° = 120 — (21 + 20 + 19 + ... + 5) = 2 so that the

above differences in the reverse order give the H versines. The rest of the contents of the verses is simple, the processes being based on the ' Rule of three '.

Verse 16. Rectification of the next H sine difference known as Bhōgya—Khanda.

The difference of the previous and the following H sine —differences being multiplied by the remaining degrees and divided by 20, the result is subtracted from the arithmetic mean of the previous and following H sine–differences to give the rectified H sine —difference, in question.

Comm. This is a formula for interpolation which agrees with the interpolation formula given by Ball in his spherical astronomy on page 18 in the form $y = yo + \dfrac{x}{h}$ $(y_1 - yo) + \dfrac{x(x-h)}{2h^2}$ $(y_2 - 2y_1 + yo)$. This formula is a re-statement of the formula enunciated by Brahmagupta in his work Brahma Sphuta Siddhānta as well as Uttara-Khandakhādya in the form " गतभोग्यखण्डकान्तरद्दळविकलग्रघात् शतैनेंग्रभिराप्तैः, तद्युतिद्दलं युतोनं भोग्याद्यूनाधिकं भोग्यम् " wherein in the place of ten–degree–interval, a fifteen–degree–interval ie 900'—interval was taken. Rule of three is a linear formula of interpolation, whereas the above is a quadratic formula reflecting much credit on the mathematical genius of Brahmagupta.

We shall now see how the formula is applied and what mathematical significance it has. Suppose it is required to find the H sine of 24° from the previous table of H sine—differences given for intervals of 10° — from the table H sin 10° = 21, H sine 20 = 41, H sine 30 = 60 where R is taken to be 120. Now to find H sin 24°, we are asked to rectify the next H sine —difference namely 19', where the table is 21, 20, 19 etc. As a first approximation, applying rule of three H sin 24° = 41 + $\frac{4}{10}$ × 19 = 43·6 = 43' − 36".

This is a crude approximation, the actual value being
48' — 48" — 14'". Application of rule of three is justified
if the H sine—differences are uniform, but they are not so,
being in a decreasing order. So, the following H sine—
difference namely 19' is to be rectified so as to be applicable
at 24°. In other words we have to take such a H sine—
difference which will hold good at 24°, not at 20° or 30° —
from 10° to 20°, the H. S. d. (H Sine —difference) is 20',
and from 20° to 30° it is 19'. If that be so what will be
exactly at 24°? If should be less than 20' and greater
than 19'. Now the argument advanced by Bhāskara is that
the H Sine —difference at the mid-point of the 2nd and 3rd

differences namely 20 and 19 should be $\frac{20 + 19}{2} = 19\cdot5$.

The H sine —difference at the end of the third interval is
19. Then by the rule of three ' If there is a decrease of
$19\cdot5 - 19 = \cdot5$, during the course of the 10° of the third
interval, what should the difference be for 4°? (where we

have to find the H sine at 24°) The result is $\frac{4}{10} \times \cdot5 = \frac{4 \times 1}{20}$

which is given by the words " यातेष्वयो: खण्डकयोर्विशेष:
शेषांशनिघ्नो नखदत् ". This decrease makes the H sine—
difference at 24°, $19\cdot5 - \frac{4}{20} = 19\cdot3$ at 24°—Now the argu-
ment to find the H sine at 24° is ' If for an interval of 10°,
the H sine—difference is 19·3, what should it be for 4°?'
The answer is $\frac{4}{10} \times 19\cdot3 = 7\cdot72$. Hence the H sine of 24° is
$21 + 20 + 7\cdot72 = 48\cdot72 = 48' - 43" - 12'"$ which is nearer the
truth 48' - 48" - 14'" than what was obtained by the crude
rule of three namely 48' - 36".

Here we have to explain Bhaskara's words more elabo-
rately, because, Kamalākara happened to criticise Bhas-
kara's words in this context. Bhaskara Says " The H sine
difference at the end of an interval is the arithmetic mean
of the preceding and succeeding differences, whereas the
succeeding one is that which holds good at end of the suc-
ceeding. In between, we have to apply the rule of three to

obtain the rectified difference." What Bhāskara means is
this. At the end of an interval, to obtain the H sine, it is
enough to add the H sine difference belonging to that inter-
val to the preceding differences. But when it is required
to find the H sine in the interior of an interval, we have to
construe that the difference at the end of the previous inter-
val is the arithmetic mean of the previous and succeeding
differences. There is apparently a self-contradiction in
Bhāskara's words; for, at the end of the interval, according
to his own words, the difference is that belonging to the
previous interval and not the arithmetic mean as postula-
ted. The contradiction will not be there when we read
Bhāskara's mind that he means "When we require to find
the H sine in the interior of an interval only, the difference
at the beginning of that interval is to be taken as the arith-
metic mean of the previous and the current differences, and
that at the end of the interval the current difference holds
good."

The truth of Bhāskara's statement could be seen
analytically as follows. The arithmetic mean of the prece-
ding and succeeding H sine differences is (The context is to
rectify the third H sine difference namely 19, for, we were
finding the H sine of $2\frac{1}{2}°$) $\dfrac{AB + BC}{2}$ (Ref fig. 8) where OA,
AB, BC etc are the successive differences.

$$\frac{AB + BC}{2} = \frac{H \sin 20 - H \sin 10 + H \sin 30 - H S 20}{2}$$

$$= \frac{H \sin 30 - H S 10}{2} = R \left(\frac{\sin 30 - S 10}{2} \right)$$

$$= R \times \cos 20 \sin 10 = \frac{H \cos 20 \, H \sin 10}{R}$$

The numerical difference of the preceding and succeed-
ing H sine differences is (i.e. यातैप्यखण्डविशेष). AB − BC
= (H sin 20 − H S 10) − (H sin 30 − H sin 20) =

$$\frac{2 \text{ H Cos } 15 \text{ H sin } 5}{R} - \frac{2 \text{ H Cos } 25 \text{ H sin } 5}{R} = \frac{2 \text{ H sin } 5}{R}$$

$$(\text{H Cos } 15 - \text{H Cos } 25) = \frac{2 \text{ H sin } 5}{R} \times \frac{2 \text{ H sin } 20 \text{ H sin } 5}{R}$$

Let now $\times$ be the point where we are to find the H sine (Here let us take it as $x°$ after the previous interval for generalisation). Then Bhaskara's formula would give

$$\frac{\text{H Cos } 20 \text{ H Sin } 10}{R} - \frac{2 \text{ H sin } 5}{R^2} \times \frac{2 \text{ H sin } 20 \text{ H sin } 5}{20} \times x$$

$$= \frac{\text{H Cos } 20 \text{ H sin } 10}{R} - \frac{x}{10 \text{ R}^2} 2 \text{ H sin } 20 \text{ H sin}^2 5$$

$$= \frac{2 \text{ H Cos } 20 \text{ H sin } 5 \text{ H Cos } 5}{R^2} - \frac{1}{R^2} \times \frac{x}{10} \times 2 \text{ H sin } 20$$

$$\text{H sin}^2 5$$

$$= \frac{2 \text{ H sin } 5}{R} \left\{ \frac{\text{H Cos } 20 \text{ H Cos } 5 - \frac{x}{10} \times \text{H sin } 20 \text{ H sin } 5}{R} \right\}$$

put now successively $x = 0°$ and $10°$ to get the rectified differences at B and C respectively; then those rectified differences would be respectively H Cos 20 H sin 10 and $\frac{2 \text{ H sin } 5}{R} \times$ H Cos 25. But $\frac{AB + BC}{2} = \frac{\text{H Cos } 20 \text{ H sin } 10}{R}$ (found above) and BC = H sin 30 − H sin 20 = $\frac{2 \text{ H Cos } 25 \text{ H sin } 5}{R}$ In other words the rectified differences at B and C are respectively what exactly has been stated by Bhāskara. Hence Kamalākara's condemnation of Bhāskara is quite unjustified.

Verse 17.   To rectify the arcual difference to obtain the arc for a given H sine.

Subtract as many H sine-differences as could be subtracted from the given H-sine. Half of the remainder multiplied by the difference of the preceding and succeeding H sine-differences and divided by the succeeding and

the result being subtracted from or added to as the case may be (added in the case of Hversines) the arithmetic mean of the preceding and succeeding H sine–differences gives the rectified H sine-difference while finding the arc for a given H sine.

Comm. Let the given H sine be that of 24° found before i.e. 48·72. We could subtract 21 and 20 from this and the remainder is 7·72; half of this is 3·86 which multiplied by (20–19) is 3·86. This divided by the succeeding difference namely 19 is ·2 approximately. The arithmetic mean of the preceding and succeeding si $\dfrac{20 + 19}{2}$ = 19·5. If the above result is subtracted from this, we have 19·5 − ·2 = 19·3. If for 19·3 we have 10° increment what shall we have for 7·72. The answer is $\dfrac{10 \times 7·72}{19·3}$ = 10 × ·4 = 4°. Hence the required arc is 20° + 4° = 24°.

The proof is analogous to the previous proof. Having subtracted 21 and 20, the remainder is 7·72, (Ref. fig. 8)

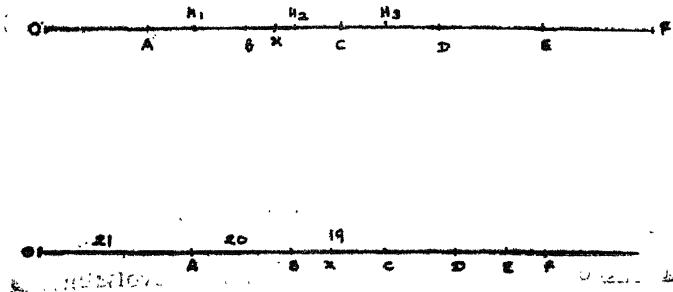Fig. 8

so that BX = 7·72. Now during the course of the succeeding interval of 19, there has been a decrease of 19·5 − 19 = ·5 or to put it in general terms, during the course of y the succeeding interval BC there has been a decrease

$\frac{x+y}{2} - y = \frac{x-y}{2}$ where x is the previous interval AB

$\frac{x+y}{2}$ is the H sine difference at B. Hence the argu-

ment is "If for y, there has been a decrease of $\frac{x-y}{2}$,

what will it be for δ (Here $\delta = 7\cdot72$)"? The answer is

$\delta \frac{(x-y)}{2} \times \frac{1}{y} = \frac{\delta}{2}(x-y) \times \frac{1}{y} = $ अद्यशेषफलघ्नम् $\times$ गते

This result is to be subtracted from
$$2$$
i.e. ·2 is to be subtracted from 19·5 to give the rectified
H sine−difference.

Verse 18. Definition of Kēndra and assignment of
sign thereto.

The excess of the longitude of the mean place over
that of the apogee or aphelion as the case may be is called
the mean anomaly. The excess of the longitude of the
point called Sīghrōccha over that of the planet rectified
by the first equation known as Manda-phala or equation
of centre is known as the Sīghra-anomaly. The equation
of centre is positive or negative according as $180 < m < 360$
or $0 < m < 180$ where m is the mean anomaly. The case
will be reverse in the case of the sign of Sīghraphala, the
second equation.

Comm. "चन्द्रसूर्यौ स्फुटौ स्यातां मान्देनैकेन कर्मणा" i.e.
The Moon and the Sun could be rectified by the equation
of centre alone. This means that the Moon revolving
round the Earth directly and that the Sun revolving rela-
tively round the Earth are subject to only one correction
namely the equation of centre for rectification. In the
case of the Sun, though the fact is that he is revolving
round the Earth relatively, assuming as Hindu astronomy
does that he is revolving directly round the Earth and
him to the correction of the equation of centre

does not alter the mathematics that goes into his rectification. According to modern astronomy the Sun and the Moon, one relatively and the other directly go round the Earth, in ellipses, the Earth being in one focus whereas in Hindu astronomy both the Sun and the Moon are taken to be going round the Earth in eccentric circles i.e. circles whose centres do not coincide with the centre of the Earth. In fact, Bhāskara says in so many words " भुमेमॅध्यं खलु भवलयस्याऽपि मध्यं यतः स्यात् , यस्मिन् वृते भ्रमति खचरो नाऽस्र मध्यं कुमध्ये, भूस्थो द्रष्टा न हि भवलये मध्यतुल्यं प्रपश्येत् , तस्मात् तद्ज्ञैः क्रियत इहतत् दोःफलमध्यखेटे i.e. The centre of the celestial sphere coincides with the centre of the Earth. The centre of the circle in which a planet goes does not coincide with the centre of the Earth. Hence an observer on the surface of the Earth finds the True planet's position differing from that of the mean planet, so that what is called the correction of Bhujaphala is to be made in the mean position of the planet to get the True position." Here it has to be noted that the Bhujaphala mentioned stands both for the equation of centre and the second equation known as S'īghraphala as well. One may wonder how it could be so, but it may be noted that in the formulation of both the equations, the centre of the eccentric does not coincide with that of the Earth and also in both the cases the equation contains the term H sine of the anomaly where the word anomaly whether it be of the first or second equation is known as ' Bhuja '.

In the case of the other planets, the fact is that they go round the Sun in elliptic orbits, the Sun being in one focus. In Hindu Astronomy, we shall see that the centre of the eccentric circle in which these planets are taken to revolve coincides with the Sun. Hence, though the ancient Hindu Astronomy postulates geocentric motion, the mathematics that goes into the formulation of the second equation, makes the Sun's centre the centre of planetary revolution. One may wonder again how the equation of centre

formulated by Hindu Astronomy agrees with its formulation in Modern Astronomy which enunciates elliptic motion; but it will be seen that the eccentric-circle theory also gives very approximately the same formula for the Equation of centre. There is just one point of difference, which does not matter. Whereas in Modern Astronomy the mean anomaly is reckoned from the perigee or perihelion as the case may be, it is reckoned in Hindu Astronomy from the apogee or aphelion. The difference is made up by prefixing the appropriate sign to the equation. The word Mandōccha stands for the apogee in the case of the Sun and the Moon and for the aphelion in the case of the other planets and the word Manda kendra stands for the mean anomaly in both the cases. In Hindu Astronomy the word 'graha' stands for not only the five planets Mercury to Saturn but also for the Sun and the Moon. Why that word is applied to the Sun and the Moon as well is, that both the Sun and the Moon also while moving among the stars along with the five other planets Mercury to Saturn, wield an influence on the residents of the Earth. The etymology of the word ' graha ' is गृह्णातीति वा गृह्यते अनेनेति वा ग्रह: ie. ' that which seizes upon the fates of the residents of the Earth', with this etymological significance only, even the lunar orbital nodes known as Rāhu and Kētu are also taken to be grahas in Hindu Astronomy. Hence translating the word graha as a planet and criticising Hindu Astronomy for taking the Sun, Moon and the lunar orbital nodes also as such is not right. In other words the translation should be pronounced wrong. Uranus, Neptune and pluto were not mentioned in Hindu Astronomy.

We shall now elucidate the eccentric and epicyclic theories of Hindu Astronomy, which will be seen to give identical position to the planets. How they came to be postulated will be also elucidated. Incidentally we deal with the ' Bhaganōpapatti ' or the proof of the numbers of sideral revolutions of the various grahas enunciated in the

beginning of the Madhyādhikāra, Bhaganādhyaya in verses
1 to 6. Even Bhāskara gave such a proof as appealed to
Āgama ie. 'Authority', which proof therefore will not be
acceptable to a student of Modern Astronomy, who is likely
to question how the Āgama came into existence.

How the Āgama came to formulate the number of
sidereal revolutions of the grahas, we shall now see.

The forefathers of Hindu Astronomy (have been re-
ported to be eighteen in number in the famous verses " सूर्यः
पितामहो व्यासो वसिष्ठोऽत्रिः पराशरः कश्यपो नारदो गर्गो मरीचिर्मनु-
रङ्गिराः लोमशः पौलिशश्चैव च्यवनो यवनो भृगुः शौनकोऽष्टादशैतेस्यु:
ज्योतिश्शास्त्रप्रवर्तकाः " Of these Brahmagupta mentions
Brahma–Siddhānta, which he reports to have resuscitated.
Varāba Mihira gave a version of the old Sūryasiddhānta,
mentioning that it accorded with observations. Āryabbata
says that he revived his system from the then existing
ocean of knowledge both good and spurious. The fact that
none of these outstanding astronomers mentioned that
they had derived their systems from a foreign source, and
the reasonableness in presuming that all these three could
not be impostors, make the author of this work feel
strongly that there should have been some works in the
name of Āgamas extant long before these Acharyās. The
argument that the crude Vedānga-jyotiṣa alone should
have existed before Āryabhata, simply because, no other
work worth the name has been discovered, may not be
correct. It is quite possible that crude works could exist
side by side with advanced scientific works, just as even
nowadays we have thinkers and works of a primitive type
existing along with highly advanced thinkers as well as
scientific works).

Bhaganopapatti. In the first place, the forefathers of
Hindu Astronomy must have noticed very easily that the
Moon has a motion among stars, for, this could be detected

even by a lay man during the course of a single night. So, the period of a single sidereal revolution, could be roughly recognized by noticing the conjunction of the Moon with a luminous star. Having thus observed a good number of sidereal revolutions, which could be done even with the naked eye, the average period could be arrived at with sufficient accuracy within the course of a few years. Having thus obtained almost accurately the average of a sidereal revolution of the Moon, the sidereal revolution of the Sun could have been arrived at as follows. The moment of an eclipse solar or lunar could be observed with the naked eye. Observing a good number of eclipses within the course of a few years, the average of a lunation could be easily arrived at very accurately, for, in between the eclipses of the same nature an integral number of lunations elapse. That the Sun also has a motion among stars must have been noticed clearly during the course of a few months, for, observing at Sunset the star that was rising, it should have been noticed gradually even during the course of a month, that the Sun must have been approaching the star or vice versa and as the stars were found to keep the distances amongst them constant, it was the Sun that was approaching the star and not the star it was that was approaching the Sun. Having decided thus that the Sun was moving among stars from west to East, the approximate period that the Sun took to complete a sidereal revolution was arrived at. Then, as both the Sun and the Moon were having east ward motion, and as the Moon has a more rapid motion, the arc by which the Moon overtakes the Sun during a day was roughly noticed. Thus arriving at a rough estimate of a lunation within which a conjunction of the Moon with the Sun recurs, it was noticed that the excess of the sidereal revolutions of the Moon over those of the Sun gave the number of lunations. Since a correct estimate of both a sidereal revolution of the Moon as well as that of a lunation were previously arrived at, the number of sidereal revolutions of the Sun during

the course of a certain period were computed whereby a correct estimate of a sidereal solar year was arrived at. This period could also be checked simultaneously by observing the interval between the heliacal risings or settings of a particular star of the Zodiac as well. Thus far, we have seen how the sidereal periods of the Sun and the Moon were determined very accurately. It may be noted that these periods as determined by the Hindu astronomers were correct to a good number of decimal places.

When once the Sun and the Moon were found to be having eastward motion among stars, and when it was discovered that there were other luminous bodies like the Jupiter and Venus etc. moving among stars, it was attempted to determine their sidereal periods. It must have been done as follows. In the first place, it was noticed that these other luminous bodies which were five in number, namely Mars, Mercury, Jupiter, Venus and Saturn, were found to be having retrograde motion also unlike the Sun and the Moon. As these five bodies were looking like stars they were named Tārā-grahas i.e. grahas looking like stars. Also a distrinction could be drawn very easily between Mercury and Venus on the one hand and the other three on the other, for, the former were always found oscillating about the Sun, never parting from him through long distances. Thus during the course of a sufficiently long interval, the geocentric sidereal periods of Mercury and Venus coincided with that of the Sun. In other words, it was taken that the geocentric sidereal periods of Mercury and Venus also were taken to be an year. This is clear from the statements made by all the Siddhāntas that in a Kalpa of 432000000 years the Sun, the Mercury and Venus all the three make 432000000 sidereal revolutions. Then with respect to the other three planets Mars, Jupiter and Saturn, it was noticed that they were having a pre-dominantly longer period of direct motion, though there was a retrograde motion for some time. This

gave the clue to arrive at an approximate estimate of their
sidereal revolutions. But the correct estimates were arri-
ved at not by observing their conjunctions with stars, for,
that would take a very long period of observation in the
case of Saturn, but, by observing a good number of their
heliacal risings or settings. The interval between two
consecutive heliacal risings or settings being a little grea-
ter than an year, ten or fifteen observations could be done
very easily by a single person. Then the aforesaid argu-
ment given in the case of finding the sidereal revolution of
the Sun, was also advanced in the case of these three
planets Mars, Jupiter and Saturn. Let $x°$ be the arc that
the planet covers during the course of a day. Let $a°$ be the
arc covered by the Sun during the same period which was
previously known and that correctly. So, during the
course of a day the Sun overtakes the planet by $(a - x)°$.
Hence to overtake 360°, the period S was computed. This
period was observed as the interval between two consecu-
tive heliacal risings or settings and known as the synodic

period. So, from the equation $\dfrac{360}{a - x} = S$, the value of x

could be arrived at, wherefrom p the sidereal period was
determined. This sidereal period could be also determined
in another way. Noting the distance covered among stars
by a planet during the course of a synodic period, using
rule of three, the sidereal period could also be arrived at
with a good accuracy, for, the retrograde motion affects
equally each synodic period. An average of such determi-
nations made in two ways could give the sidereal periods
of Mars, Jupiter and Saturn with a good amount of accu-
racy. It will be noted here, that the average of a good
number of geocentric sidereal periods in the case of these
planets (called Superior) is also the heliocentric sidereal
period (for a proof of this statement reference may be made
to page 80 of the author's *A critical etudy of the ancient
Hindu astronomy*, published by the Karnatak University
Dharwar). It is why the sidereal periods of these planets

as given by Hindu astronomy tally with the heliocentric sidereal periods given by modern astronomy. This is also one of the reasons why heliocentric motion of the planets could not be detected by Hindu astronomy, and also why a statement was made that "In the case of Mercury and Venus the Sun was the planet, and they are termed as S'ighrōcchas, whereas in the case of Mars, Jupiter and Saturn, they are the planets while the Sun plays the part of S'ighrōccha "कुजजीवशनीनां तु रवि: शीघ्रोच्चनामकः, ब्रशुक्रयो: तौ शीघ्रनामकौ" (An elucidation of this statement will be given shortly)

In the case of the planets Mercury and Venus (Inferior planets) one may wonder how under the geocentric theory, their heliocentric periods could be arrived at, though they were not recognized as such but were pronounced as the " geocentric periods (not considered as heliocentric)" of two points known as their S'ighrōcchas. Here we come across the peculiar concept of a S'ighrōccha which arose out of the fact of postulating a geocentric system in the place of the heliocentric. This concept is to be elaborated, in as much as confusion is there in the minds of many interpreters of Hindu astronomy in this behalf.

In the first place let us consider as to how the rectification of the Sun and the Moon, known as sphutīkaraṇa was achieved. Having got their sidereal periods, their mean daily motions were calculated. Also a period was conceived, during which this Sun and the Moon would perform an integral number of revolutions. This period was termed as a Mahāyuga (or simply a yuga as we hereafter name it) whose duration was estimated as 4320000 solar years. That the yuga is an integral L.C.M. so to say of the sidereal periods of the Sun and the Moon (also of the other planets as we shall see shortly) could be seen from the statement of the Surya Siddhānta युगे र्थंब्रशुक्राणं

खचतुष्करदार्णवाः, कुजार्किगुरुशीघ्राणां भगणाः पूर्वयायिनाम् , इन्दो रसाग्नित्रित्रीषुसप्त भूधरमार्गणाः " i,e. 'In a yuga, the Sun the Mercury and Venus perform 432000 sidereal revolutions as well as the S'īghrōcchas of Mars, Jupiter and Saturn, whereas the Moon performs 57753336 revolutions' (It may be recalled here that the Mercury and Venus are oscillating about the mean position of the Sun; also it will be noticed that the Sun playing the part of the S'īghrōcchas in the case of Mars, Jupiter and Saturn, their S'īghrōcchas are also deemed as making the same number of revolutions as the Sun. In as much as the S'īghrōcchas in the case of Mercury and Venus are looked upon as different from the planets, so in the case of Mars, Jupiter and Saturn also. the Sīghrocchas are taken as different Divine entities though coinciding with the Sun in position). However smaller periods could be conceived as integral L.C.M's of the sidereal revolutions of the Sun and the Moon, but a presumption sponsored by a sense of orderliness in the Cosmos, that the planets should all have been started from the Zero point of the Zodiac, made the integral L.C.M. to be of such a dimension as 4320000 solar years in which period the other planets also would have made an integral number of sidereal revolutions. Here in this point the traditional Hindu astronomers place their faith in the Āgama, which said that the planets were all started at the Zero-point of the Zodiac in the beginning of the yuga and were ordained to return to the same point at the close of the yuga. Even a rational astronomer like Bhāskara, apparently placing faith in the Āgama, while adducing a proof in the name of Bhaganōpapatti, states that after obtaining the mean daily motions of the planets, calculates them for the period of a Kalpa taking it on trust that the planets were started at the initial point of the Zodiac in the begin-

ning of the Kalpa, A modern astronomer, however, questions the assumption that the planets were all started at the first point of the Zodiac, and even though they might all have been in conjunction at that point in some remote past, whether it was the initial point of the reported Kalpa. Proceeding on the basis of the reported initial conjunction of all the planets at the first point of the Zodiac, and calculating the number of days that have elapsed from the beginning of the yuga, the mean positions of the Sun and the Moon were computed. Noticing that these mean positions did not exactly accord with the true observed positions, the ancient astronomers tabulated the differences between those mean and true positions. These differences were found to be zero at two diametrically opposite points, and maximum roughly at two points differing by a quadrant from them. To account for these differences, the thought that occured to their minds was that probably the Sun and Moon did not move in a circle whose centre coincided with that of the Earth but were moving in an eccentric circle i.e. a circle whose centre is at some other point than the Earth's centre. This surmise could be made because unequal motion was accountable only on varying distance from the Earth's centre and a celestial body appearing to move fastest must be nearest whereas the same appearing to move slowest must be farthest. Thus in the first place seeing no reason for non-circular motions and also expecting the celestial bodies to move only in circles, for, a circular motion appealed to them as the most ideal motion, the ancient astronomers later postulated an eccentric circular motion with respect to the Sun and the Moon. This postulation appeared to give good results as seen below.
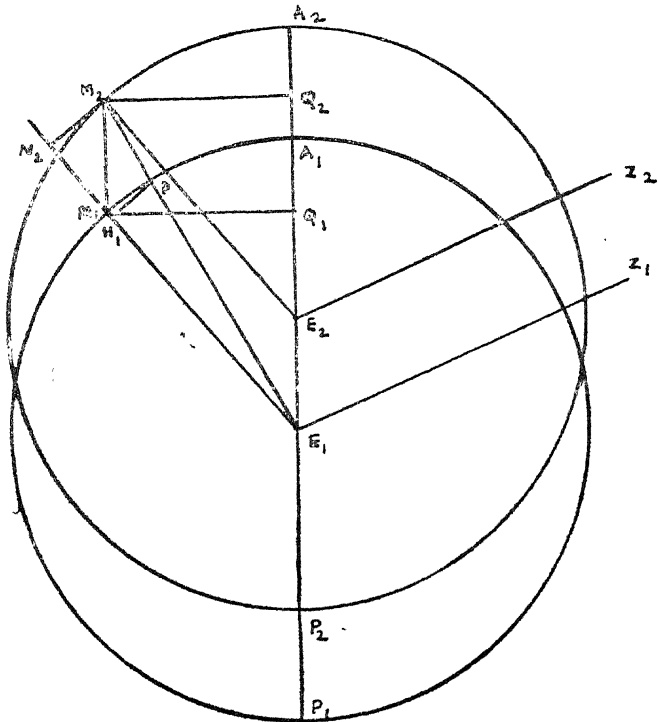
Fig. 9

Eccentric circle theory.—Let E_1 be the earth's centre; let M_1PA_1 be the circular orbit in which the planet (here the Sun or the Moon) moves with a uniform motion. This planet is termed the Madhyagraha or the mean planet. Let $M_2A_2P_2$ be the actual orbit of the planet whose centre E_2 is removed a little away from E_1. Since the centre E_2 is moved in a vertical direction away from E_1, every point of the eccentric circle (E_2) will be vertically over the corresponding point of the mean circle. Thus M_2 will be the position of the actual planet where M_2 is vertically above M_1 the mean planet. Join E_1M_2 to cut the mean circle in P. Since A_2 is the position of the actual planet farthest from E_1 the Earth's centre, the planet should have

the slowest motion there. So this point A_2 is termed Mandoccha, Manda because it is point where the planet is slowest and Uccha because it is the highest or the farthest point from the Earth's centre. Corresponding to this Mandoccha in the eccentric circle A_1 is termed the Uccha in the mean circle. Also p the point where E_1M_2 the line joining the Earth's centre to the actual planet and called the Mandakarna, cuts the mean orbit is taken to be the position where the apparent planet is situated. Thus 'p' is seen to be deflected from the mean planet M_1 towards the Mandoccha on which account the Mandoccha is considered to be attracting the planet ' — गेह्याःकर्षंको भवति " as Bhāskara puts it. The angle $A_2\widehat{E_2}M_2$ is spoken of as the Manda-Kendra or the mean anomaly and it is equal to $Z_2\widehat{E_2}M_2 - Z_2\widehat{E_2}A_2 = $ longitude of the planet minus the longitude of the Mandoccha, where E_1Z_1 and the parallel E_2Z_2 are directions towards the Zero-Point of the Zodiac. This accounts for the statement ' मद्दुच्चेन हीनो ग्रहो मन्दकेन्द्रम् ' (of the verse under elucidation) i.e. the excess of the longitude of the planet over that of the Mandoccha is termed Mandakendra. While M_1 is termed the Madhya-graha in the mean orbit, M_2 is termed the prativritta-Madhyagraha, and not spaṣṭagraha as might be deemed, while p is spoken of as the spaṣṭagraha or the True planet or apparent position of the planet. The word Prativritta stands for the eccentric circle. Now M_1P the difference between the mean and True positions is spoken of as the Mandaphala which corresponds to the modern 'Equation of Centre'. To find its value draw perpendiculars PN_1 and M_2N_2 on

Triangles $E_2M_2Q_2$ and $M_1M_2N_2$ are evidently similar Hence $\dfrac{M_2Q_2}{E_2M_2} = \dfrac{M_2N_2}{M_1M_2}$ so that $M_2N_2 = \dfrac{M_1M_2}{E_2M_2} \times M_2Q_2 = $

$\dfrac{r}{R}$ H sin $M_2\widehat{E_2}A_2$ (1) (which is equal to r Sin $\widehat{E_2}$ in modern terms) Now in the case of the Equation of centre which is

generally a small quantity M_2N_2 is taken to be equal to PN_1. If, however, this approximation is not made, $PN_1 = \dfrac{M_1N_2 \times E_1P}{E_1 M_2}$ (by the similarity of the triangles $E_1 PN_1$ and $E_1 M_2 N_2$) so that the actual equation of centre is $\dfrac{r}{R}$ H sin $E_2 \times \dfrac{R}{K} = \dfrac{r}{R}$ H Sin E_2 where $K =$ Mandakarna $E_1 M_2$. As M_2 moves from A_2 to P_2, the equation of centre as given by (1) gradually increases from Zero to a maximum r when $E_2 = 90°$ and decreases from this maximum to Zero when $E_2 = 180°$. Thus from what was noticed from the tabulated differences between the computed mean positions and observed true positions, the fact that those differences vanish at A_2 as well as P_2 the diametrically opposite point of the Mandoccha (not called S'īghroccha, for this word S'īghroccha will be seen to have altogether a different connotation) was verified. The maximum value of $M_2 N_2$ 'r' is termed the Antyaphala-jya or the H sine of the maximum equation of centre from which the arc could be found. In the case of the Sun and the Moon from the maximum differences between the computed and observed positions, their H sines were found and taken to be equal to 'r' in the respective cases. From this value of r, the circumferences of the circles whose radius equals r, were found and termed as Mandaparidhis. Why the circumferences were found is that in all positions of M_1M_2, the value of $M_1M_2 = r$ (in as much as the corresponding points of the two circles will be as much distant as the centres of the circles from each other so that $E_1E_2 = M_1M_2 = $ Constant $= r$) so that M_2 will always lie on a circle whose centre is M_2 and radius r. This circle is known as the Manda-NichocchaVritta or an epicycle, where the word Nicha stands for the P_2 which is nearest the earth, and the compound word Manda-Nichoccha-Vritta means that circle which makes the planet occupy the Nicha and the Uccha points; the word Manda pertains to the Manda-phala or the equation of centre in

contradistinction to the word S'īghra which we shall shortly deal with.

In modern astronomy the equation of centre is given approximately to be equal to 2e sin m where 'm' stands for the mandakendra so that $r = 2e$. It will be shortly seen from a subsequent table that this formulation of the equation of centre gives results which closely accord with their modern values. The true or apparent positions of the Sun and the Moon could be obtained fairly well from the above formulation, so that it is stated that चन्द्रसर्यों स्फुटौ स्यातो मान्देनैकेनकर्मणा i.e., the Moon and the Sun could be rectified by the equation of centre alone." This is quite in order for, the Sun and the Moon may be taken to be going round the Earth in ellipses, with the earth in one focus, the former relatively and the latter directly.

After having formulated the method of rectification in the case of the Sun and the Moon, the next question was with respect to the Tārā grahas i.e. Mercury, Venus and Mars, Jupiter and Saturn. As these are going round the Sun and the Sun going round the earth relatively, the process of rectification got complicated. In the first instance, the ancient astronomers must have tabulated the differences between the mean computed positions and the observed true positions. In the case of Mercury and Venus, the case appealed different from what it was in the case of the other three planets, for the simple reason that the mean positions of the former were taken to coincide with the mean Sun. This meant that for rectification, the elongation had to be computed and added to or subtracted from the mean position of the Sun to get the apparent geocentric positions of Mercury and Venus. The analogy of the method of the formulation of the Mandaphala is taken here also by imagining (1) eccentric circular motion and (2) postulating an Uccha. In the case of the Mandaphala, the equation was zero when the mean planet coincided with the Mandoccha. Here the equation is zero when

elongation is zero, i.e. when the apparent geocentric position of the planet coincides with the Sun, who is taken to be the mean planet. Naturally therefore the Uccha is taken to coincide with the Sun the mean planet, when the planet is in conjunction with the Sun. The maximum equation was had in the case of the Mandaphala when the arc between the Uccha and the mean planet was a right angle. So, here also, the maximum equation i.e. the maximum elongation should be had when the Uccha is a at right angle from the Sun. Thus an Uccha was postulated with the following criteria namely (1) It should be a point moving in a geocentric circle (2) It should coincide in direction with the Sun when the planet is in conjunction with the Sun (3) It should be removed by a right angle from the Sun when the elongation is maximum (3) It should be removed from the Sun by 180°, again when the planet coincides in direction with the Sun (4) It should have a longitude exceeding that of the Sun by 270 when again the elongation is a maximum on the other side and finally (5) It should complete a circle with respect to the Sun when again the planet coincides in direction with the Sun.

When such a point was conceived it is clear that this Uccha is not the same as the planet, as some have mis construed, because while the planet oscillates about the Sun by a particular angle (29° in the case of Mercury and 45° in that of Venus) in Uccha completes a circle with respect to the Sun and further as Hindu Astronomy postulated geocentric motion, the Uccha is a point construed as going in a geocentric circle. By the above postulation the synodic period of the Uccha is equal to the period of oscillation of the planet about the Sun. But the latter period is no other than the synodic period of the planet so that the synodic periods of the Uccha and the planet coinciding their sidereal periods should be equal. In other words the Uccha so conceived is a point other than the planet going round in a geocentric circle and having a geocentric side-

real period equal to the heliocentric sidereal period which again means that the geocentric longitude of the Uccha is ,the heliocentric longitude of the Planet. Thus the radius vector to the planet from the Sun is parallel to the geocentric radius vector of the Uccha. This accounts as to how the heliocentric sidereal periods of Mercury and Venus could be found under a geocentric concept and also as to how the heliocentric planets are themselves spoken of as their respective Ucchas, while their mean planet is the same as the Sun. On this count it was mentioned by the Hindu Astronomers ज्ञशुक्रयोः ग्रहः सूर्यो भवेत् तौ शीघ्रनाकौ ie. The mean Planet of Mercury and Venus is the Sun himself where as they are themselves spoken of as their Ucchas. The phrase ' they are themselves ' in the above statement is significant as it connotes that the word 'they' stands for the heliocentric planets, though it was not stated in so many words. Shortly we shall see also that the centre of the eccentric circle coincides with the centre of the Sun also and applying Bhāskara's statement 'यस्मिन्वृत्ते भ्रमति खचरो नाऽस्य मध्यं कुमध्ये' i.e. the centre of the circle in which the planet moves does not coincide with that of the Earth', the Sun was, though unwittingly taken as the centre of the Planetary motion. Thus we see how even the geocentric postulation also could help computation of the Planetary positions, the mathematics behind revealing heliocentric motion. What Copernicus achieved was that he identified that the point about which the planets revolved which was construed by ‚the Hindu astronomers as an imaginary point not coinciding with the earth's centre, was no other than the Sun himself.

In the case of Mercury and Venus the so-called Sīghra-phala came to be discovered first and we shall presently see why their elongation was called Sīghra-phala and how the Uccha postulated as above came to be termed Sīghroccha. Since initially the equation was to be zero, when the Planet and the Uccha coincided with the Sun

and then the elongation has to increase as the Uccha gained over the Sun, the initial conjunction was the modern Superior conjunction. The other position of the Uccha when again the elongation i'e. the equation is Zero must be therefore the Inferior conjunction. Also at the motion of Superior conjunction, the planet must be having the maximum daily motion, as it is clear from a heliocentric figure that at that point the relative motion of the planet with respect to a geocentric observer is the sum of the velocities of the planet and the earth. Hence this Uccha is spoken of as the Sīghroccha also because the Uccha being a geocentrically moving point having heliocentric angular motion, its velocity is always greater than that of its planet namely the Sun. The word Uccha is applied because at the Superior conjunction the planet is farthest or highest from the earth. The excess of the longitude of this Uccha over the longitude of the planet ie. the Sun is known as the Sīghrakendra or anomaly as it is said in the verse under commentary 'चलोच्चं ग्रहोनं भवेत् शीघ्रकेन्द्रम्'. Thus in the case of Mercury and Venus, the Sīghraphala came to be discovered first. This being discovered, formulated as will be shortly shown, and applied to the mean Sun as the planet, still it was found that there was a difference between the computed position and the observed position. Such differences were tabulated. By analogy from the case of the Sun and the Moon, it was thought that there should be also a Mandoccha here also, so that the point indicated by the position of the mean planet after being corrected by the equation where the above tabulated difference was zero, was identified as the Mandoccha. From the H Sine of the maximum difference taken as the radius of the Manda epicycle, its circumference was then computed.

In the case of the superior planets, we have already said that the geocentric sidereal periods accord with their heliocentric ones. Calculating the mean position of the and finding its difference from the observed true

position, such differences were tabulated. It was discovered that these difference almost vanished when the planet was in conjunction with the Sun and attained a maximum when the elongation was nearly a right angle from an analogy from the Manda-Karma i.e. process of obtaining the Manda-phala. Since the differences attained their maximum value when the elongations were nearly a right angle it could be seen that the Sun played the part of the Uccha in this case. As the Sun has a quicker motion than the planet and also as at conjunction the planet has the quickest motion relative to the Earth while it is farthest from the Earth the Uccha ie the Sun here, is termed S'ighroccha. The excess of the longitude of the Sun over that of the planet is termed accordingly the S'ighra-kendra and the S'ighra-phala the equation was formulated as will be shown. Applying this S'ighra-phala to the mean position, the differences still found between the position so obtained and the observed true position were tabulated. The point indicated by the above position where the difference was found to be zero, was identified as the Mandoccha, and through the maximum difference, the Mandaparidhi was formulated.

In the above discourse, we have tried to give an account of how the originators of Hindu Astronomy could give us a workable system. We never assumed that an Āgama gave us the numbers of sidereal revolutions of the planets or the measures of the epicycles Manda or S'ighra. But in the explanation given by Bhāskara under Bhagaṇōpapatti, one will notice that when Bhāskara gave the proof of the Moon's sidereal revolutions, he said that having got the true positions of the Moon on two consecutive days, the mean positions were computed from the true by an inverse process of applying the equation of centre, and getting the mean daily motion of the Moon from those mean positions, the number of sidereal revolutions in a Kalpa were obtained. Here the Upapatti or the proof adduced by Bhāskara was not a proof but only a verification in as much as (1) he assumed the formulation

of the Mandaphala from the Āgama without pointing out
how it was formulated and (2) he assumed the period of a
Kalpa and that at the beginning of the Kalpa the planets
were all in conjunction at the first point of Aries. Similarly
in the Upapatti adduced by Bhāskara with respect to the
Mandocchas of the planets, he assumed the formulation of
S'īghra-phala on the basis of Āgama without proving how
the concept of S'īghra-phala was arrived at by the founders
of Astronomy and how the difference between the observed
apparent positions and the computed mean positions, was
resolved into two equations the Mandapha'a and the S'īghra-
phala. In the proof adduced with respect to the S'īghroccha
of Mercury and Venus also Bhāskara did not mention
anything as to how their heliocentric sidereal periods could
be obtained but simply assumed the formulation of the
Mandaphala and S'īghra-phala as already being there on
the basis of Āgama.

We shall now proceed to describe the method of formu-
lation of the S'īghra phala with respect to the five Tara-
grahas, star planets namely Mercury, Venus and Mars,
Jupiter, Saturn and show how so different a set of geomet-
ries of the ancients and the moderns the one geocentric
and the other heliocentric could give identical formulation
with respect to S'īghra-phala. Let us consider the case of
Mercury and Venus in the first instance.

Having taken the mean Sun to play the part of the
'Graha' in the case of Mercury and Venus and having
formed a concept of S'īghroccha as mentioned before, whose
geocentric period of revolution was determined, without
suspecting it to be the heliocentric sidereal period of the
planet the ancient Hindu astronomers assumed by analogy
from the case of Mandaphala with respect to the Sun and
Moon, that the centre of the circle in which these planets
revolve does not coincide with the centre of the Earth. In
other words, they continued eccentric circle theory here
so that without suspecting heliocentric

their mathematics led to them to make the centre of the eccentric circle coincide with the Sun himself. On this basis alone it was given to Copernicus to formulate heliocentric theory, sponsored by a thought that the Heavenly Sun could not be deemed as a satellite of the 'Mundane' Earth.

(Ref. fig. 9). The same figure 9 will also serve the purpose to obtain the S'ighraphala, only M_1 M_2 will be now on the right hand side of the S'ighrocchas A_1 A_2, for, the latter will be taken to be in advance of the mean planets Kaksha-Vrittīya Madhyagraha M_1 (i.e. mean planet of the deferent) and prati-Vrittīya Madhyagraha M_2 (i.e. mean planet of the eccentric). The points A_1 add A_2 are themselves called the Kaksha-Vrittīya S'ighrōccha and prati-Vrittīya S'ighrōccha respectively. As was shown in the case of the Mandaphala from the eccentric figure 9, M_2 N_2 =

$$\frac{r}{R} \text{ H sin (Kendra) so that } PN_1 = \frac{r}{R} \text{ H sin (Kendra)} \times \frac{R}{K}$$

$$= \frac{r}{K} \text{ H sin (Kendra)} = \frac{\text{Antyaphalajya} \times \text{Sīghrakendrajyā}}{\text{Sīghrakarna}}.$$

We shall take this for elucidation in the appropriate context.

Verse 19. Three Rasis each of 30° constitute a quadrant, and there are four quadrants in a circle which are respectively odd, even, odd and even. In the odd quadrants the Kendra covered is itself called Bhuja whereas in the even ones, the complement thereof is called Bhuja. Also, the complement of the Bhuja is called the Koti.

Verse 20. R — H sine = Co. H versine and R — H cosine = H versine and R — Co. H versine = H sine, R — H versine = H cosine.

Verse 21. Also $\sqrt{R^2 - H \sin e^2}$ = H cosine, = H sine. Similarly

Dyujya, $\sqrt{R^2 - \text{Dyujya}^2}$ = Krāntijya; $\sqrt{R^2 - \text{Drig-jyā}^2}$

= S'anku, $\sqrt{R^2 - S'anku^2}$ = Drig-jyā. In all the cases cited above, the radins R happens to be the hypotenuse.

Comm. The convention in verse 19 corresponds to saying in modern trigonometry that sin $\overline{90 + \theta}$ = cos θ sin $(180 - \theta)$ sin θ, sin $\overline{180 + \theta}$ = $-$ sin θ, sin $\overline{270 - \theta}$ = $-$ cos θ sin $\overline{270 + \theta}$ = $-$ cos θ, sin $\overline{360 - \theta}$ = $-$ sin θ. In Hindu trigonometry the sign is understood and not explicitty mentioned.

Krāntijyā, Dyujyā, Drig-jyā and S'anku, are respectively H sin δ, H cos δ, H sin Z, H cos Z where δ is declination and Z the Zenith-distance of a celestial body. Taking the radius of the celertial equator to be R, the radius of the diurnal circle of a celestial would be equal to R $\times$ cos δ = H cos δ which is called Dyujyā because it is the radius of the diurnal circle.

Verse 22. The lengths of the circumferences of the Manda—epicycles are respectively 13° – 40', 31° – 36', 70°, 38°, 33°, 50,* for the Sun, Moon, Mars, Mercury, Jupiter, Venus and Saturn.

Comm. Bhāskara has given these measures reportedly on the basis of Āgama or ancient authority. The peculiarity of measuring the circumferences in degrees less than 360°, is due to the idea that these circumstances are measured in relation to that of the deferent or Kakshā Vritta taken to be 360°. In other words, circumference of the epicycle of a planet as given above : circumference of the mean orbit : : x : 360 = radius of the epicycle : radius of the mean orbit where x is the measure of the circumference of the planetary epicycle. It may be mentioned once again that the radius of a planetary epicycle is the measure of the greatest

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* The printed book of Brahma Sphuta Siddhānta gives in the case of Saturn 30° only which might have been the mistake of the scribe (Vide verse 36, Spashtādhikāra B. S.). In the place of शून्यरामा: it ought to have been

equation of centre pertaining to the planet which may be taken to be equal to 2e as a first approximation where e is the eccentricity of the elliptic orbit of the planet.

It may be further mentioned here that in Sūrya-siddhānta, as well as elsewhere in this work, the circumferences are given to vary continuously. This variability curiously achieves ellipticity in the orbit as may be seen as follows.

In the case of the Sun, the epicycle has a periphery of 14° when m = 0 or 180° and of $13\frac{2}{3}°$ when m = 90° or 270 according to Sūryasiddhānta, where m is the Manda-kendra or mean anomaly. At any arbitrary point, where

Fig. 10

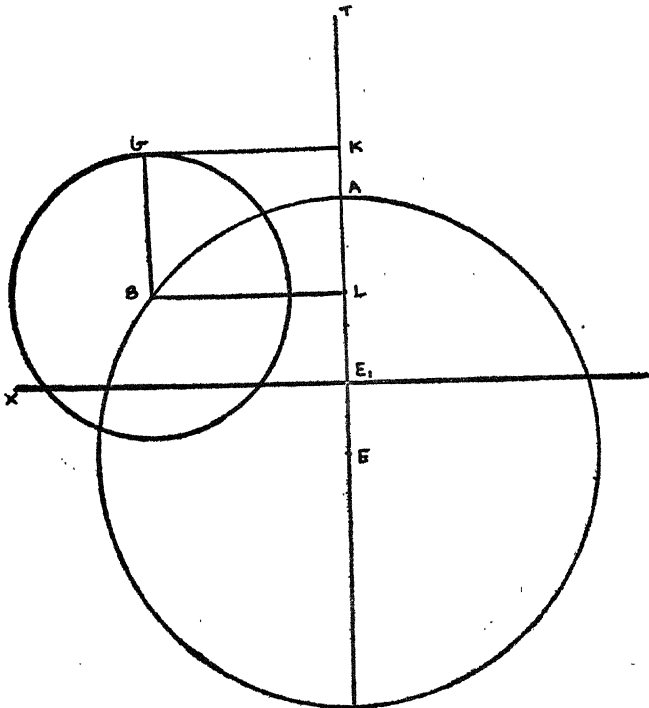the mean anomaly is m the periphery is given to be

$14° - \dfrac{20' \text{ H sin m.}}{R}$ The corresponding radius will there-

fore be $r - \dfrac{20.}{2\pi} \quad \dfrac{\text{H sin m}}{R} = r - \text{H sin m}$ (say) where $r =$

$\dfrac{14°.}{2\pi}$ (Ref. fig. 10) Let c be the earth's centre, A the posi-
tion of the apogee $EE_1 =$ the radius of the epicycle meas-
ured along cA, B any arbitrary position of the mean planet
and b the position of the planet in the epicycle. Here the
radius Bb is not equal to the max. radius equal to EE_1 i.e.
r but equal to $r - \text{H sin m}$. Take E_1 as the origin and
E_1A as the y—axis and a perpendicular to E_1A through E_1
namely E_1x as the positive direction of the X-axis. If the
mean anomaly BEA be m, then the coordinates of the true
planet are given by x = BL = H sin m (1) y = E_1L +
Bb = EL — EE_1 + r — λ H sin m = H cos m — r + (r —
λ H sin m = H cos m — λx.

$\therefore$ y + λx = H cos m   (2) Squaring and adding I
and II $x^2 + (y + \lambda x)^2 = \text{H sin}^2 m + \text{H cos}^2 m = R^2$

$\therefore$ $x^2 (1 + \lambda^2) + 2\lambda xy + y^2 = R^2$ which is an ellipse
with centre E_1

Verses 23, 24, 25. The peripheries of S'ighra epicycles.
The peripheries of the epicycles of the star-planets Mars,
Mercury, Jupiter, Venus and Saturn are respectively
243°-40′, 132°, 68°, 258° and 40°. The H sine of the Manda
mean anomaly of Venus being multiplied by 2 and divided
by 343, and the result being subtracted from the periphery
gives the rectified Manda periphery. The H sine of its
S'ighra mean anomaly being multiplied by 5 and divided
by R, and the result being added to the S'ighra periphery
gives the rectified periphery. The smaller of the H sine or
H cosine of the S'ighra anomaly of Mars being multiplied
by $6\frac{2}{3}$ and divided by H sin 45° and the result in degrees
being subtracted from or added to as the case may be,

the aphelion gives the rectified aphelion. The S'īghra periphery being reduced by the above degrees gives the rectified S'īghra periphery in case the S'īghra anomaly is 90° < m < 180 or 270° < m < 360°.

Comm. In the commentary Bhāskara adds that in the case of Venus the Mandaperiphery of 11° as given is at the end of even quadrants whereas at the end of odd quadrants it is 9°, wherefore the enunciated rectification. Similarly in the case of his S'īghraphala, the periphery of 245° mentioned is at the end of even quadrants whereas at the end of odd quadrants it is 263°, and so the suggested rectification. Again in the case of Mars, the aphelion as computed is the same at the end of all quadrants whereas in the middle of the quadrants it is to be increased or decreased by $6\frac{2}{3}$° when the anomaly is as stated. Also in the case of this Mars, the S'īghra periphery mentioned is at the ends of quadrants. In the middle of the quadrants the periphery is to be reduced as suggested. In all these interpolations Bhāskara accepts the Āgama as enunciated by Brahmagupta.

We shall deal with the geometrical nature of these S'īghra peripheries shortly in the appropriate place.

Verse 26. To obtain what are called Bhujaphala and Kotiphala both in the case of Mandaphala as well as S'īghraphala. The H sine and H cosine of the Manda or S'īghra anomalies multiplied by the respective peripheries and divided by 360°, or multiplied by r and divided by R gives the Bhujaphala or Kotiphala where r and R are respectively the radius of the Manda or S'īghra peripheries and R the radius of the deferent taken to be 3438′. If the radius 3438′ be respectively multiplied by the Manda or S'īghra peripheries and divided by 360°, the result will be the H sine of the maximum Mandaphala or S'īghraphala, known as in either case.

Comm. As per the formulation.

$$\text{Bhujaphala} = \frac{\text{H sin m} \times c}{360} = \frac{\text{H sin m} \times r}{\text{R}}$$

$$\text{Kotiphala} = \frac{\text{H cos m} \times c}{360} = \frac{\text{H cos m} \times r}{\text{R}}$$

in the case of Mandaphala or S'īghraphala where c = periphery of the Manda or S'īghra periphery, r=Antyaphalajyā defined above R = 3438′ and m stands for the Manda or S'īghra anomaly. These Bhujaphala and Kotiphala will be used in their respective contexts.

Verses 27, 28, 29. Calculation of what is known as S'īghrakarṇa.

$$(\text{H Cos m} \pm \text{r})^2 + \text{H Sin}^2 \text{ m} = \text{K}^2 \text{ (1)}$$

$$(\text{R} \pm \text{Kotiphala})^2 + \text{Bhujaphala}^2 = \text{K}^2 \text{ (2)}$$

$$\text{R}^2 + \text{r}^2 \pm 2\,\text{R} \times \text{Kotiphala} = \text{K}^2 \text{ (3)}$$

$$\text{R}^2 + \text{r}^2 \pm 2\,\text{r} \times \text{H Cos m} = \text{K}^2 \text{ (4)}$$

The ·arc of the H Sine of the equation of centre is called the Mandaphala.

Comm. Ref. fig. 9. From triangle $E_1M_1M_2$ $(E_1M_1 + M_1N_2)^2 + M_1N_1^2 = E_1M_2^2 = K^2$. But $M_1N_2 = $ Kotiphala and $M_2N_2 = $ Bhujaphala defined previously so that we have the second formula for K enunciated above. From the similarity of the triangles $E_1M_1Q_1$ and $M_1M_2N_2$ we have $\dfrac{M_2N_2}{M_1Q_1} = \dfrac{M_1M_2}{E_1M_1}$ so that $M_2N_2 = \dfrac{r}{R} \times M_1Q_1$; Since M_1Q_1 is called the Bhuja, the corresponding M_2N_2 in the Antyaphalajyā triangle is called the Bhujaphala. Similarly M_1N_2 is called the Kotiphala.

Again $(E_1Q_1 + Q_1Q_2)^2 + M_2Q_2^2 = E_1M_2^2$ where $E_1Q_1 = $ H Cos m, $Q_1Q_2 = M_1N_2 = r$ and $M_2Q_2 = M_1Q_1 = $ H Sin m. From this we have the first formula enunciated.

Again expanding $(E_1M_1 + M_1N_2)^2$, $(E_1M_1 + M_1N_2)^2$ $+ M_2N_1^2 = E_1M_2^2$ we have the third formula; similarly expanding $(E_1Q_1 + Q_1Q_2)^2$, $(E_1Q_1 + Q_1Q_2)^2 + M_2Q_2^2 = E_1M_2^2$ we have the fourth formula.

Since the S'īghraphala has been defined to be equal to $\frac{r}{K}$ H Sin m, we have had the necessity of knowing the value of K. The convention of signs mentioned in the formulation in the words 'योगो मृगादावय कर्कटादौ केन्द्रेऽन्तरम्' is due to the fact that cosine is positive in the fourth and first Quadrants and that the Kotiphala becomes negative in the 2nd and 3rd Quadrants as could be seen by drawing the figure in those Quadrants.

Now we shall prove what is most important, namely that postulating an entirely different geocentric motion how the Hindu Astronomers could formulate the S'īghraphala which accords exactly with the heliocentric theory, assuming of course coplanar circular orbits. Let figures 11 and 12 pertain to the modern heliocentric geometry,
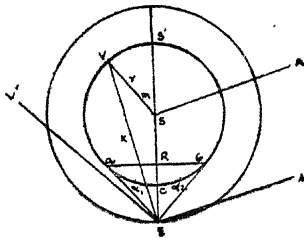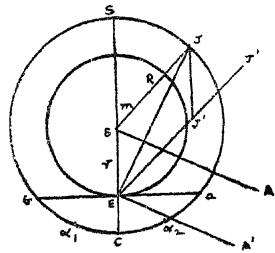

Fig. 11   Fig. 12

the former with respect to the Inferior planets Mercury and Venus signified by V, and the latter to the superior planets Mars, Jupiter and Saturn signified by J. Let fig. 13 pertain to the Hindu geocentric geometry dealing with both the Inferior and Superior planets as well. In the heliocentric figures let S = Sun, E = Earth, SA = directon to ुि, the Zero-point of the Zodiac from the Sun, Ⅳ^{A1}

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geocentric direction towards Aswini. Let SV, EV¹, be the heliocentric and geocentric directions of the S'īghroccha where V is the actual planet and V¹ an imaginary point. Draw EJ' 11 to SJ. Let the radius of the inner and outer heliocentric circles be respectively r and R. Let K be the radius vector to the planet in both the figures.

Let in fig. 13, E_1=Earth's centre, E_2= the centre of the eccentric circle which we shall presently show to be coinciding with the Sun's position. Let M_1, M_2 represent the mean planets in the deferent and the eccentric known as Kaksha-Vrittīya Madhyagraha and prati-Vrittīya Madhyagraha. Let A_1, A_2 be the S'īghrocchas in the deferent and the eccentric. Let $E, E_2 = r$ known as Antyaphalajyā and R the radius of both the deferent and the eccentric. Let K be the radius vector to the planet known as S'īghra-Karṇa.

We shall prove that m the S'īghra anomaly of the Hindu figure will be the same as 'm' as marked in the heliocentric figures. S'īghra anomaly is defined as longitude of S'īghroccha — longitude of the $l_1 - aE_1M_2 = a^1\widehat{E_2}A_2$ $a^1\widehat{E_2M_2} = m$ fig. 13. In the heliocentric fig. 11, since A^1EV^1 is the longitude of the S'īghroccha, and $A^1\widehat{E}S$ the longitude of the Sun treated as the Madhyagraha of the Inferior planet $A^1\widehat{EV^1} - A^1\widehat{ES} = V^1\widehat{ES} = V\widehat{SS^1} = m =$ the S'īghra anomaly. In fig 12, $m = S^1\widehat{SJ} = S\widehat{EJ} = A^1\widehat{ES} - A^1\widehat{EJ^1} = A^1ES -$ $A\widehat{S}J =$ Longitude of the Sun treated as the S'īghroccha of the Superior planet minus longitude of the heliocentric planet known as Mandasphutagraha or the planet rectified for the Mandaphala or equation of centre = S'īghra anomaly In the case of the Inferior planet the heliocentric direction of the planet is equal to the S'īghroccha. Now consider the triangles ESV, JSE, $E_1M_1M_2$ of the three figures. Evidently $E\widehat{S}V = J\widehat{S}E = E_1\widehat{M_1}M_2 = 180 - m =$ Supplement of S'īghra anomaly. If, further it is shown and (it will be shown subsequently) $\dfrac{SV}{SE} = \dfrac{SE}{SJ} = \dfrac{M_1M_2}{}$ the similarity of the triangle E_1M_1M geocentric figure separately with ESV and JSV will have been established. Taking this similarity to have been established, $M_1\widehat{E}M_2$ known as S'īghraphala will be equal to $S\widehat{EV}$ in fig. 11 and $S\widehat{JE}$ in fig. 12. In fig. 13, M_2 the prativritta Madhagraha is also known as the pāramārthikagraha or the actual planet where as p its geocentric position on the Kakshāmandala is taken to be the true planet or apparent position of the planet. In figures 11 and 12, EV and EJ are the directions to the true planets V and J so that the angles between the Sphutagraha and the Mādhyagraha (ie the Manda Sphutagraha =

(in fig 11) and $= A^1EJ$ -
S'ighraphala. Once the similarity of the triangles fig. 12 is established, the equality of the S'īghraphala will be established. Also due to the similarity mentioned above the formulae for K as given in Hindu Astronomy should also accord with that in the heliocentric figures. In fact in the heliocentric figures $K^2 = R^2 + r^2 + 2Rr$ cosm $= R^2 + r^2 + 2RH$ cosm which is indentical with the four formulae given before as per verses 27, 28, 29. It will be seen that the epicycle (M_1) with radius M_1M_2 will be identical with the inner circles in the heliocentric circles, whereas the Kakshamandal (E_1) with radius R will be identical with the outer circles of the heliocentric figures.

Before we proceed further, we shall annex the table wherein the ratio r/R as given in Hindu Astronomy will be seen to accord with that in modern astronomy.

Planet	Periphery of the S'īghra-epicycle	Periphery of the deferent	Ratio	Value in modern astronomy taking Earth's radius to be unity
Mercury	132°	360°	$\frac{132}{360} = \cdot37$	·387
Venus	258°	360°	·716	·723
Mars	$243\frac{2}{3}°$	363°	1·5	1·52
Jupiter	68°	360°	5·3	5·2
Saturn	40°	360°	9	9·5

In the light of this table the similarity of the triangles ESV$_1$ and JSE with E$_1$M$_1$M$_2$ is now established.

Formula for Síghraphala from the heliocentric figures

In fig. 11, $\dfrac{r}{K} = \dfrac{\sin \widehat{SEV}}{\sin \widehat{ESV}}$ so

$$\dfrac{r}{K} \times \sin\ m$$

In fig. 12 $\dfrac{r}{K} = \dfrac{\sin \widehat{SJE}}{\sin \widehat{ESJ}}$ so that $\sin \widehat{S\,J\,E} = \dfrac{r}{K} \sin\ m$

Both these accord with the Hindu formula.

It will be interesting to point out here that in fig 11, keeping the earth constant and supposing the Sun S to go in a circle with centre E and radius ES, the orbit of the Inferior planet V will play the part of the epicycle of Hindu Astronomy. Thus in the case of the Inferior planets, the epicyclic theory is only a different version of the heliocentric theory. In the case of the Superior planets, however, (fig. 12) cut off $EJ^1 = SJ$ along EJ'' parallel to SJ; then J^1J will be parallel to ES just as M_1M_2 is parallel to E_1E_2 in fig. 13. Then the circle with E as centre and EJ^1 as radius corresponds to the deferent of fig 13, whereas the circle (J^1) with centre J^1 and radius J^1J corresponds to the epicycle. The circle with S as centre and radius SJ corresponds to the eccentric.

Verse 30. The equation of centre pertaining to the Sun, and the Moon using a simpler table of H sines where the radius = 120 units. The H sines of the mean anomaly as found from the simpler H sine table where radius = 120, multiplied by 20, and divided by 1103 and 477 respectively gives the equation of centre of the Sun and the Moon in degrees.

Comm. The maximum equation of centre with respect to the Sun is 2°–10′–31″. Then the argument is " If by the H sine of the anomaly equal to the radius 120, we have the above max. equation what shall we have for H sin m ?".

The answer is $\dfrac{\text{H sin m} \times 2°\text{-}10'\text{-}31''}{120} = \dfrac{2\frac{21}{120}°}{120} \times \text{H sin m}$

very approximately $= \dfrac{261}{14400} \text{ H sin m} = \dfrac{20}{1103} \text{ H sin m}$

Similarly in the case of the Moon, the maximum equation of centre is $5°\text{-}2'\text{-}8''$. By the same argument as above we have $\dfrac{\text{H sin m} \times 1133}{225 \times 120} = \dfrac{\text{H sin m} \times 20}{\frac{54000}{1133}} = \dfrac{\text{H sin m} \times 20}{477}$

Verse 31. Rectification of the mean daily motion of the Sun and the Moon.

The H cosine of the mean anomaly divided by 54 in the case of the Sun and in the case of the Moon multiplied by 4 and divided by 7 gives the increment or decrement in the respective mean motions according as $90 < m < 270$ or $270 < m < 360 + 90$.

Comm. We have Equation of centre $= \dfrac{r}{R} \text{ H sin m} =$ E (say) so that differentiating $\delta E = \dfrac{r}{R} \text{ H cos m} \dfrac{\delta m}{R}$. But $\dfrac{r}{R} \text{ H cos m}$ is called kotiphala and δm is called Kendra gati so that $\delta E = \dfrac{\text{Kotiphala} \times \text{Kendragati}}{R}$. Since Kotiphala is negative when $90 < m < 270$ δE is negative but in Hindu Astronomy we measure the Kendra not from perigee as in modern astronomy but from aphelion so that the equation of centre is strictly $- \dfrac{r}{R} \text{ H sin m}$ if sign is also taken into consideration. Hence δE must be $+$ ve. Since $M + E = S$ where M is the mean planet, E the equation of centre and S the true planet $\delta S = \delta m + \delta E$ so that the true motion is equal to the mean motion plus δE. As δE is $+$ ve when $90 < m < 270$ as mentioned above we have to add this to the mean motion to get the true motion. This

is called gatiphala or what is to be added to the mean motion to give the true motion. Incidentally we have commented on the contents of verse 37.

Verse 32. The S'īghra phala with respect to the Star-planets.

$\dfrac{H \sin m \times r}{R}$ being multiplied by the radius R or the product of H sin m and r being divided by K the arc of the result gives the S'īghraphala.

Comm. In the verse it is mentioned that the product of the Bhujaphala and the radius is divided by K so that the formula for the Bhujaphala being $\dfrac{H \sin m \times r}{R}$ the S'īghraphalajya will be $\dfrac{H \sin m \times r}{K}$ which is stated in the alternative. We have already derived this formula before where we got $H \sin E_2 = \dfrac{H \sin m \times r}{K}$. The arc of this will be E_2 i.e. the S'īghra-phala, (E_2 because we take E_1 as the Mandaphala).

Verses 33 and 34. An alternative formulation of S'īghra-phala.

H sin m being multiplied by R and divided by K and the difference between the arc of the result and H sin m will be the S'īghra-phala. Here H sin m belongs to the eccentric. The arc of the maximum S'īghra-phalajya added to or subtracted from 90° will give respectively the Quadrants and the H sine will have to be taken of the elapsed Kendra or its Koti according as the Quadrant is odd or even.

Comm. Ref. fig. 13. From the similarity of the triangles $E_1 M_2 N$ and $E_1 PM$,

$$= \dfrac{R}{K} \times M_2 N$$

.But PM is the H sine of m^1 where m^1 is called Sphuta kendra (m is called the madhya-Kendra). Hence

$$H \sin m^1 = \frac{R}{K} \times M_2N = \frac{R}{K} \times Bhujajy\bar{a} \text{ (in the eccentric)}$$

$$\therefore m^1 = H \sin^{-1}\left(\frac{R}{K} \times Bhujajy\bar{a}\right) = PA_1$$

$\therefore M_1P = S'\bar{\imath}ghraphala = M_1 A_1 - PA_1 = $ Kakshya-mandala Bahu minus the chāpa m^1 — Here a clear under-standing of the word Bāhu or what is the same Bhuja should be had. The arc pertaining to the angle m in the eccentric is known as the Bāhu in the eccentric and that to the angle m in the deferent as the Bāhu in the deferent. When $0 < m < 90$, m is itself spoken of as Bāhu; when $90 < m < 180$, $180 - m$ is spoken of as the Bāhu; when $180 < m < 270$, $m - 180$ is the Bāhu and when $270 < m < 360$, $360 - m$ is the Bāhu. Thus the Bāhu is that angle whose H sine will be H sin m numerically. When it is said in the verse ' त्रिज्याहता कर्णह्रता भुजज्या ' the word Bhuja is the arc M_2A_2 as is mentioned in the same verse ' ज्ञेयोऽत्र बाहुः प्रतिमण्डलस्य '. The second part of the verse divides the eccentric circle into such quadrants that in them S'īghraphala increases from Zero to a maximum, decreases again from a maximum to zero, again increases from zero to a maximum and again decreases from a maximum to zero. Thus at A_2 of the eccentric the S'īghraphala is zero; at a_1, it is a maximum namely the arc b_1c_1 where $H \sin b_1c_1 = a_1c_1 = r$; Thus in the course of A_2a_1 the arc of the eccentric the S'īghraphala gradually increases from zero to a maximum and in the course of a_1a_2 the S'īghraphala decreases from a max to zero. Again from a_2 to a_3 it increases from zero to a max and from a_3 to A_2 it decreases from the maximum to zero. Thus the quadrants in the case of S'īghraphala are not of 90° but arcs A_2a_1, a_1a_2, a_2a_3 and a_3A_2 which are respectively of mag-nitude $90° + H \sin^{-1}r$, $90 - H \sin^{-1}r$, $90 - H \sin^{-1}r$ and $90 + H \sin^{-1}r$. In the case of Mandaphala also, the quadrants should have been of the same magnitude if the so-called Karnānupāta has been postulated i.e. reducing the Manda-

phala from the extremity of the Karna to the extremity of
the radius in the deferent; but as this Karnānupāta is not
adopted, the difference being negligible the quadrants are
all of equal magnitude i.e. each of 90°.

In the course of the commentary of this verse Bhāskara
mentions that for Mercury, as the maximum S'īghraphala is
21°-31'-43", the quadrants are of magnitude 3-21-31-43,
2-8-28-17, 2-8-28-17 and 3-21-31-43 respectively.

Also in the commentary Bhāskara adds that a_1 which
is the point of intersection of the eccentric with the hori-
zontal diameter E_1b_1 the S'īghraphala is maximum and
that at that point the mean motion is itself the true motion
गति: स्पष्टा परं

. That the S'īghraphala at a_1 and a_3 is maximum is
clear from the figure 13, where it is equal to the arcs b_1c_1 and
b_2c_2 whose H sine is equal to r. To prove that the mean
motion is itself the true motion, we have the equation
$M_2 + E_2 = S$ where M_2 is the mean planet here or the
Mandasphutagraha or planet rectified for the equation of
centre, (by the equation $M_1+E_1=M_2$, M_1 being the original
mean planet and E_1 the equation of centre) so that $\delta M_2+\delta E_2$
$=\delta S$ where δM_2 is the mean motion here, δS the true
motion and δM_2 is the variation in the S'īghraphala; but at
a_1 and a_3 E_2 the S'īghraphala being maximum δE_2 is zero.
Hence $\delta M_2 = \delta S$ which means that the mean motion is
itself the true motion.

Verse 34. Latter half, 35 and 36 former half.

The mean planet rectified for the equation of centre or
Manda-phala is called Mandasphuta. Then subtracting the
longitude of the Mandasphuta from that of the respective
S'īghroccha, the result will be the S'īghra anomaly from
which the S'īghra-phala is to be obtained. Rectifying the
Mandasphuta for this second equation namely S'īghraphala,
again obtaining therefrom the equation of centre effecting-

this in the original mean planet and again correcting for
S'ighraphala and repeating the process till a constant value
is obtained, the true planet is had with respect to the star-
planets other than Mars. But with respect to M·rs, let
first the mean planet be corrected for half of the equation
of centre. Then make half of the correction of S'ighraphala.
Take the resulting planet to be the mean planet and again
finding the equation of centre, make this whole correction
in the original mean planet. Again taking the resulting
planet to be the Mandasphuta effect the entire S'ighraphala.
Then we have the true planet.

Comm. The Suryasiddhānta stipulates the same kind
of correction in the case of all the star-planets.

दं
द्ये फलार्धं प्रथमं ततो
ग्रद्धे ॥" i.e. In the first place half of the S'ighra-
phala is to be effected in the mean planet; taking that to
be the mean planet and computing the equation of centre
half of it is administered; taking the resulting to be the
mean planet and computing the equation of centre, the
entire equation of centre is now to be administered in the
original mean planet; taking the result to be the Manda-
sphutagraha, and computing the S'ighraphala, it is to be
administered in full in the Mandasphuta. Then we have
the true planet.

In modern astronomy, the equation of centre is first
done and the result will be the planet in its heliocentric
elliptic orbit. To reduce it to the geocentric position, the
second correction is made which corresponds to the Hindu
S'ighraphala. Thus the two corrections administered suc-
cessively gives the apparent or true geocentric planet.

Though the mutual relationship as conceived between
the equation of centre and the S'ighraphala in Hindu Astro-
nomy is deemed irrational by modern interpreters of Hindu
tronomy, there is some rationale in the process as ex-

plained by this author in his work 'A critical study of
Ancient Hindu Astronomy' (published by the Karnatak
University) page 98.

Verse. Cited from Golādhyāya.

The equation of centre is to be applied to the mean
planet to obtain the centre of the Sīghraepicycle; then to
obtain the true position the Sīghraphala is to applied to the
Mandasphutagraha; hence the two equations are mutually
related so that the true position is obtained after repeated
application of the two equations.

Comm. Explained above.

Verse 36 latter half and 37. The true daily motion
of the planet is the excess of the longitude of the true planet
of the next day over that of the true planet of the previous
day.

The Kotiphala being multiplied by the daily motion of
the Manda mean anomaly and divided by the radius, and
the result being added to or subtracted from the mean
motion, gives what is called Mandasphutagati.

Comm. Already explained before. If M_1, M_2 and S be
the mean planet, Mandasphutagraha, and the true planet
respectively, and if E_1 and E_2 be respectively the two equa-
tions, then

$$M_1 + E_1 = M_2, \quad M_2 + E_2 = S \text{ so that}$$

Here $\delta M_1 =$ mean daily motion, $\delta E_1 =$ daily variation
in the Mandaphala, $\delta M_2 =$ daily motion of the Mandasphuta
graha or what is the same Mandasphutagati, $\delta E_2 =$ daily
variation in the second equation and $\delta S =$ True daily
motion.

Lallāchārya formulates the Mandasphutagati in a different manner which is equally correct (Ref. verse 45 Spaṣṭādhikāra Siṣyadhī Vriddhida ''त्रिज्याहता ग्रहगति: मृदुकर्णभक्ता मन्दस्फुटा भवति '' i. e. $\dfrac{\delta m \times R}{K}$ = Mandasphuta-gati where $\delta m =$ Madhyagati or mean motion and K is the Manda Karṇa equal to $\sqrt{R^2 + r^2 \pm 2 R r \cos m}$

$\therefore$ Mandasphutagati =

$$\sqrt{\dfrac{\delta m \times R}{1 \pm 2 \dfrac{r}{R} \cos m + \dfrac{r^2}{R^2}}} = \delta m \left(1 \pm \dfrac{2r}{R} \cos m \right)^{-\frac{1}{2}}$$

$= \delta m \left(1 \mp \dfrac{r}{R} \cos m \right)$ neglecting the smaller term

$-\dfrac{r^2}{2R^2}$ within brackets

$= \delta m \mp \dfrac{r H \cos m \, \delta m}{R^2} \, \delta m = \delta m \mp \dfrac{r H \cos m}{R} \times \dfrac{\delta m}{R}$

$= \delta m \mp \dfrac{\text{Kotiphala} \times \text{Mandakendragati}}{}$ as given by

Bhāskara.

Verse 38. In the case of the Moon, obtaining the true Moon for a particular moment and his daily motion for the day, the ending moment of the tithi near at hand is to be computed with that daily motion, and the method of successive approximations is to be used to rectify the ending moment. In the case of the ending moment of the tithi being sufficiently far away, then it does not matter even if the above daily motion is applied to get the approximate ending moment. In as much as the Moon's daily motion is great and varies from moment to moment the motion at the moment is to be used.

Comm. Strictly speaking the ending moment of every tithi is to be computed by the method of successive approximation. That is why in the computation of eclipses,

Moon's hourly motion is given in modern almanacs. Since it is very cumbrous to use the method of successive approximation to determine the ending moment of every tithi, the Hindu almanac-makers generally compute the ending moment of a tithi using the daily motion of the moon computed for the moment of Sun-rise of the day. Only for ritual purposes, the method of successive approximation is used and also in the computation of eclipses.

Verse 39. Computation of the S'īghragatiphala.

$$\frac{H \sin (90 - E_2) \times \delta m}{K} = \delta S \text{ where } \delta l \text{ is the daily}$$

motion of the S'īghrōccha, E_2 = S'īghraphala, δm = daily motion in the S'īghra mean anomaly, K = S'īghrakarṇa, and δS the true motion of the planet. If δS is negative the planet is retrograde.

Comm. This is mathematically an important verse, and the proof given by Bhāskara really reflects his genius. Before we attend to his proof, we shall give this a modern treatment. (Ref. figs. 14, 15). SA and EA¹ are the
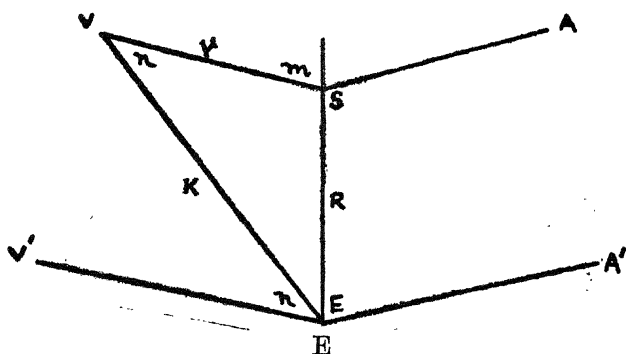
Fig. 14

heliocentric and geocentric directions to Aswini the Hindu Zero-point of the Zodiac; S = Sun, E = Earth; V = Inferior planet Venus or Mercury; J = Superior planet; E = S'īghra-phala, K = S'īghra Karṇa; M = S'īghra anomaly.
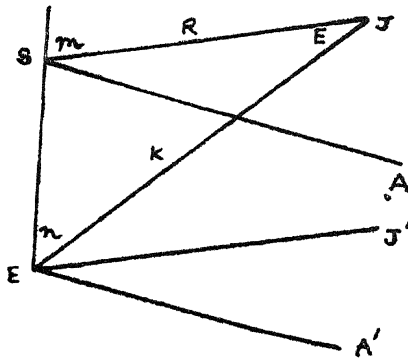
Fig. 15

True motion of the planets $= \delta \, (A^1 EV)$ or $\delta \, (A^1 EJ)$

But $\delta \, (A^1 EV) = \delta \, (A^1 EV^1 \stackrel{\frown}{-n})$ and $\delta \, (A^1 EJ) = \delta \, (A^1 ES \stackrel{\frown}{-n})$

In the case of the Inferior planet $\delta \, (A^1 EV^1) = \delta \, (ASV)$ = Sīghrōcchagati and δn is Sphutakendragati where n is called Sphutakendra, m being called Madhyakendra. In the case of the Superior planet $\delta \, (A^1 ES) = $ Sīghrōcchagati because the Sun plays the part of Sīghrōccha in the case of a Superior planet and $\delta n = $ Sphutakendragati as before. Hence in both the cases, Sphutagati=Sīghragati—Sphuta-kendragati. We have now to find Sphutakendragati to obtain Sphutagati, as Bhāskara remarks rightly "महामतिम-रेव स्पष्टीकृता". In other words we have to find δn. From the figures K cosn — R cos m $= r$ (i) Differentiating this we have — K sin n δn + cos n δK + R sin m δm = 0 (ii). But $K^2 = R^2 + r^2 + 2Rr$ cos m so that $2 \delta K \times K$ = $- 2Rr$ sin mδ m (3) Eliminating δK between (2) and

(3) — K sin nδ n — $\qquad\qquad$ + R sin

i.e. K sin nδn = R sin mδm $\left(1- \dfrac{r}{K} \, \cos n \right)$

$$- \dfrac{\text{R sin}}{\text{K}} \qquad (\text{K} - r \, \cos$$

But $K - r \cos n = R \cos E$.

$\therefore \quad \delta n = \dfrac{R \sin m \delta m \times R \cos E}{K^2 \quad n}$ But $R \sin m = K \sin n$

Cancelling $\delta n = \dfrac{R \cos E \delta m}{K} = \dfrac{H \cos E \, \delta m}{K}$ as given by

Bhāskara. In the formula $H \sin E = \dfrac{r \, H \sin m}{K}$, Bhāskara

perceived the variability of both $H \sin m$ and K on the right hand side and he exclaims "न हि केन्द्रगतिजमेव फलयोरन्तरं स्यात् , किन्त्वन्यथाऽपि अद्यतनभुजफलश्वस्तनभुजफलान्तरे त्रिज्यागुणे अद्यतनकर्णाहृते यादृशं फलं न तादृशं श्वस्तनकर्णाहृते, स्वल्पान्तरेऽपि कर्णे भाज्यस्य बहुत्वात् बह्वन्तरं स्यादित्येतदानयनं हित्वा अन्यत् महामतिमद्भिः कल्पितम् , तद्यथा केन्द्रगतिरेव स्पष्टीकृता " i.e. " The variation in the S'īghraphala is not entirely constituted by the variation in m but also by that in K......So leaving the method of seeking δE through the formula $H \sin E = \dfrac{r \, H \sin m}{K}$ the great intelligent astronomers used the formula.

Sphutabhukti = S'īghra Bhukti — Sphuta Kendrabhukti (Bhukti means gati) wherein it was sought to obtain the variation in Sphutakendra i.e. $\overset{\frown}{n}$ in the figures.

We have given a proof of Bhāskara's formula, which circumvented finding S'īghragatiphala but which sought directly Sphutabhukti, by using the modern heliocentric figures. We shall now see how Bhāskara could deal with such a tough problem. Refer fig 16. Let P_1, P_2 be two positions of the planet on two consecutive days relative to the S'īghra A_2, so that $P_1 E_1 P_2$ is the Sphuta Kendragati spoken of. It will be noted that it is not Sphutagati because P_1, P_2 are positions of the planet relative to A_2 which is itself moving (as rightly remarked by Bhāskara). It is Sphuta Kendragati because $A_2 E_1 P_1$ and $A_2 E_1 P_2$ are the
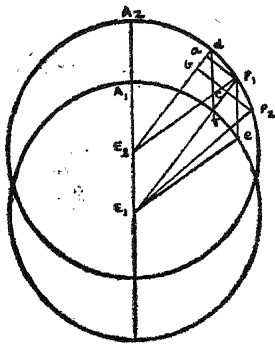
Fig. 16

Kendras on two consecutive days whereas Madhyakendragati is $P_1\widehat{E_2}P_2$. Also, we have the equation.

S'ighra — Sphutagraha = Sphutakendra so that S'ighragati — Sphutagati = Sphutakendragati. Hence Sphutagati = S'ighragati — Sphutakendragati. So we have now to seek the value of $P_1\widehat{E_1}P_2$. Let P_1a stand for the S'ighraphala of the first day which is equal to e f, f being the true planet of the first day. E_2 d will be parallel to E_1 f because P_1 e being parallel to E_1 E_2 and e f being equal to P_1 d, d f 11 P_1 e (parallels to E_1 E_2 cut off equal arcs on the two circles). This may be seen also as follows. Since P_1 d is taken to be equal to e f, e being the mean planet and f the true on the first day e $\widehat{E_1}$ f = P_1 $\widehat{E_2}$ d. But E_1 e 11 E_2 P_1 ∴ e E_1 f = E_1 $\widehat{P_1}$ E_2 ∴ P_1 $\widehat{E_2}$ d = E_1 $\widehat{P_1}$ E_2 and alternate angles being equal E_1 P_1 11 E_2d. P_1 a is the H sine of P_1 d where P_2 b is the H sine P_2 d. Looking upon P_1P_2 as an increment in P_1 d i.e. looking upon the Kendragati P_1 $\widehat{E_2}$ P_2 as an increment in S'ighraphala, Bhāskara uses the method of Bhogyakhanda sphuti Karana to obtain the Sphutakendragati. From the figure.

$$P_2c = P_2b - P_1a = H \sin (G + \delta m) - H \sin E$$

$$= \frac{H \sin E \; H \cos \delta m + H \cos E \; H \sin \delta m}{R} - H \sin E$$

Taking $H \cos \delta M = R$ and $H \sin \delta m =$

$P_2 c = \dfrac{H \cos E \, \delta m}{R}$ This is at the end of $E_1 P_2$ i.e.

at the end of K; so to get the corresponding chord in the deferent we do Karnānupāta so that the result is $\dfrac{H \cos E \, \delta m}{R} \times \dfrac{R}{K} = \dfrac{H \cos E \, \delta m}{K}$ as given by Bhāskara. We have cut short Bhāskara's method of Bhōgyakhanda Sphutīkaraṇa to make it clear to a modern student. Since $P_2 c$ is small, passing on from the H sine to the arc is not necessary, for, the H sine of a small arc is equal to the arc itself.

Bhāskara's argument, however, is as follows:—"If for 225, we have Bhōgya Khanda, what for δm?" The result is $\dfrac{B \times \delta m}{225}$; Then B is rectified as follows:—" When the H cosine E is equal to the radius, i.e. initially in the H sine table, the Bhōgyakhanda is 225, then what is it for $H \cos E$?" The result is $\dfrac{H \cos E \times 225}{R}$ which we have to substitute for B. Then if this be at the end of K what is it at the end of R?" The result is $\dfrac{H \cos E \times 225}{R} \times \dfrac{\delta m}{225} \times R = H \cos E \times \delta m$

Before we proceed to explain ''
we shall explain what Bhāskara pointed out as a mistake in Lallācharya.

Verse 40. Let Mathematicians understand that what formula was given by Lallācharya for S'īghragatiphala is not correct. When the anomaly is 90° or 270°, the gatiphala vanishes and there will be gatiphala at the points where it ought to be Zero according to his formula.

Comm. Ref. verse 45, Spaṣṭādhikāra, Siṣya Dhī-vṛddhida

i.e.

$$\frac{)\times R}{K} \times \frac{\text{Śīghraphala Bhōgyakanda}}{225}$$

= Śīghragatiphala.

Here the quantity within the brackets is δm. What Lallācharya had in his mind is as follows. ' If for the Ādya Khanda 225 we have H cos E = R, what shall we have for the Bhōgya Khanda of the Śīghraphala ? " The result would be $\left[H \sin (E + 225) - H \sin E \right] \times \frac{R}{225} =$ $\left(\frac{H \sin E \ H \cos 225 + H \cos E \ H \sin 225 - H \sin E}{R} \right) \times \frac{R}{225}$ Taking H cos 225 = R and H sin 225 = 225 it would be $\frac{H \cos E \times 225}{225} \times \frac{R}{225} =$ H cos E. So Lallācharya's

formula would become $\frac{O\overset{m}{\underset{}{}}}{K} \times$ H cos E as given by Bhās-kara. The charge levelled at Lallācharya is due to the fact that by the word ' Āśu·chāpa ' by which Lallācharya meant Bhāskara meant ' आशुकेन्द्रचाप ' . When the Kendra = 90 or 270, the Bhōgyakhanda being Zero, the Śīghragatiphala would be Zero. Also where it ought to be zero namely (90 + H sin'*r*) & (270 − H sin'*r*) it would not be zero. If Lallācharya had really meant what Bhāskara allributed to him, the formula would be $\frac{H \cos m\delta m}{K}$

it is very unlikely that Lallācharya would have meant this wrong formula, for, even if K were taken by him to be steady, $\delta \left(\frac{r \ H \sin m}{K} \right) = \frac{r \ H \cos m\delta m}{K}$ and Lallāchar-ya's formula does not contain '*r*'. The only non-rigorous part in Lallācharya's formula is at the point where he took

H cos 225 = R and H sin 225 = 225 which is rather crude. Bhāskara of course improved on this crudeness by taking to be an increment in E.

In the commentary under this verse, Bhāskara, having misinterpreted Lallācharya's phrase आशुवाप as meaning आशुकेन्द्रवाप and not as आशुफलचाप which was in the mind of Lallācharya, goes on recounting examples where the wrong formula attributed to him would give wrong results. So, we need not enter into those details.

Now we shall correlate Bhāskara's formula with its modern counterpart. Assuming coplanar heliocentric circular orbits for planets, let us see at what points two planets appear mutually stationary, that is, have a Zero relative angular velocity before they appear mutually retrograde.

Let S = Sun, E = Earth, J = Jupiter, u = Earth's linear velocity, v = Jupiter's linear velocity r and R the orbital radii of the Earth and Jupiter respectively. Let EE¹ and JJ¹ be perpendiculars to EJ so that when the relative velocity of Jupiter with respect to the Earth perpendicular to EJ is Zero, Jupiter will appear stationary as seen from the Earth. This
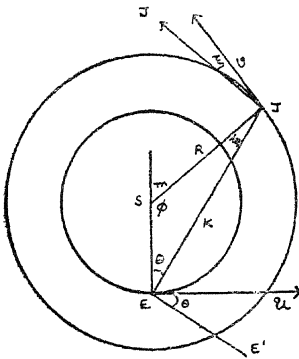
Fig. 17

means that u cos θ + v cos ξ = 0   I

$$\therefore \quad \frac{u}{v} = \frac{-\cos\xi}{\cos\theta} \qquad \text{II}$$

But from triangle ESJ,

R cos ϕ + K cos θ = r III and

r cos ϕ + K cos ξ = R   IV

From III & IV $\quad \dfrac{\cos \xi}{\cos \theta} = \dfrac{r \cos \phi - R}{R \cos \phi - r} \qquad$ V

Equating $\dfrac{\cos \xi}{\cos \theta} \quad$ from II and V

$$- \frac{u}{v} = \frac{r \cos \phi - R}{R \cos \phi - r} \quad \text{so that} \quad \frac{ru + Rv}{rv + Ru} = \cos \phi$$

If m be the S'īghra anomaly $\quad m = 180 - \phi$

so that $\cos m = -\left(\dfrac{ru + Rv}{rv + Ru}\right) \qquad$ VI

Now we shall show that Bhāskara's formula accords with

this $\quad$ Spaṣṭagati $=$ S'īghragati $- \dfrac{H \cos E \, \delta m}{K}$. As per

$$\text{Bhāskara}$$

$$\text{Spaṣṭagati} = 0 \text{ if S'īghragati} = \frac{H \cos E \, \delta m}{K} \qquad \text{VII}$$

ie. Jupiter appears stationary as seen from Earth, if S'īghra-

gati $= \dfrac{H \cos E \, \delta m}{K}$

The angular velocities of Earth and Jupiter are respect-

ively $\dfrac{u}{r}$ and $\dfrac{v}{R}$ so that the Sun's apparent velocity is also

$\dfrac{u}{r}$ and $\delta m =$ Kēndra gati $=$ Sun's apparent velocity

minus Jupiter's heliocentric velocity $= \dfrac{u}{r} - \dfrac{v}{R}$

$\therefore$ Substituting in VII

$$\text{S'īghragati} = \frac{u}{r} = \frac{H \cos E}{K} \left(\frac{u}{r} - \frac{v}{R} \right)$$

$$\therefore \frac{u}{r} \left(\frac{H \cos E}{K} - 1 \right) = \frac{H \cos E}{K} \times \frac{v}{R}$$

ie. $\dfrac{u}{r} \left(\dfrac{H \cos E - K}{K} \right) = \dfrac{H \cos E}{K} \times \dfrac{v}{R}$

But $H \cos E = R \cos E = R \cos \xi$ (here) ξ standing for E and $R \cos \xi - K = -r \cos \theta$

so that $\dfrac{u}{r} \times - r \cos \theta = u \cos \xi$

$$\therefore \quad u \cos \theta + u \cos \xi = 0$$

This is equation I derived from modern methods, so that Bhāskara's formula accords with I and the rest follows as before i.e. $\cos m = - \left(\dfrac{ru + Rv}{rv + Ru} \right)$

Substituting the values of r and R for each of the planets and noting that $r = $ epicyclic radius or Antyaphalajyā $R = 3438'$, $u = $ mean velocity of the Sun and $v = $ mean velocity of the planet, we have the respective values of m when planets appear stationary or, what is the same we could more easily use equation I noting that $\theta = m - \xi$ and thereby get the values of m for stationary values.

We shall give here some points of observation pertaining to S'īghraphala and S'īghragatiphala.

We have $M_2 + E_2 = S$ I where $M_2 = $ Mandasphutagraha and $E_2 = $ the S'īghraphala where from we have $\delta M_2 + \delta E_2 = \delta S$ II i.e. Mandasphutagati + S'īghragati phala = Spastagati (a) Let E_2 be maximum so that $\delta E_2 = 0$, then $\delta M_2 = \delta S$. This means that in the heliocentric figures 11 and 12, at the points a and b, the S'īghraphala being maximum, the Mandasphuta gati will be itself the Spashtagati. This line ab, it will be seen corresponds with the so called "कक्षायध्यगतितिर्यप्रेखा्प्रतिवृत्तसम्पातरेखा" i.e. the line cutting the eccentric, drawn through the centre of the deferent.

(b) The planet begins to retrograde only after the Spashtagati vanishes i.e. after $\delta M_2 + \delta E_2$ becomes Zero. Taking δM_2 to be almost a constant since the Mandagati phala is small, the negative value of δE_2 must cancel δM_2 in order that the Spashtagati may be zero. δE_2 becomes

negative when the planet courses the arc of the smaller segment ab, because E_2 decreases from a maximum value to zero. As a matter of fact δE_2 negatively increases along ac and again increases from a negative minum at c to Zero at b as we course along cb (vide fig. 11 & 12). Thus the planet will assume zero velocity at two points symmetrical about c and in between ab. In other words the planet will be retrograde along the arc d_1 cd_2 not entirely along ab as some have misconstrued. Regarding E_2, it is clear that it is zero at S'. then gradually increases to a maximum as the planet traces S'a, the maximum being assumed at a, then it decreases from the maximum to zero as the planet courses the arc ac. and then increases from zero to a maximum at b and finally decreases from that maximum to zero at S' again. Keeping the Earth constant it will be noted that an Inferior planet always goes anticlockwise whereas a Superior planet always goes clockwise. Also it will be seen that the S'ighraphala is positive as the planet courses the arc S'ac, whereas it is negative as it courses cbS'.

(c) The values of the spashtagati at S' and C will be respectively putting H cos E = R in the formula.

$$\text{Spashtagati} = \text{S'ighragati} - \frac{H \cos E \delta m}{K} \; . \; \text{Here for}^\bullet$$

S'ighragati we may put U and U − V for δm, and putting K = R + r at S' and R − r at C, the values of the spashtagati would be

$$\frac{U - R(U - V)}{R + r} \quad \text{and} \quad \frac{U - R(U - V)}{R - r} \quad \text{ie.} \quad \frac{RV + ru}{R + r},$$

$$\frac{RV - rU}{R - r} \text{ respectively.}$$

$$\frac{rU}{r} > \frac{RV - rU}{R - r} \text{ if } R^2V - rRV + RrU - r^2U >$$

$$R^2V - RrU + rRV - r^2U$$

ie. if rR (U — V) > Rr (V — U).   U — V is + ve and equal to V — U.

So the positive velocity at S' of the planet will be equal to its negative or retrograde velocity at C.   A quantity assuming values K, O, — K,O,K must have a value numerically less than K in between.   Thus the velocity direct or retrograde at any point of the orbit is less than the numerical value of the velocity at S' and C.

Verse 41.   Retrograde motion.

The planets Mars, Mercury, Jupiter, Venus and Saturn will be retrograde when the S'īghra anomaly assumes values 163, 145, 125, 165 and 113 respectively and the direct motion again ensues at (360—163), (360—145), (360—125), 360 — (165) and (360 — 113) respectively.

Comm.   Since we have had the formula VI

$$\cos m = - \left(\frac{r\text{U} + \text{R}v}{r\text{V} + \text{RU}} \right) \text{ for stationary points,}$$

noting that $\cos (180 - \theta) = - \cos \theta = \cos (180 + \theta)$ the stationary points are symmetrically situated with respect to S' of figures 11 and 12.   As mentioned before substituting the values of U, V, r, R for all the planets we can prove the veracity of Bhāskara's statement.

Verse 42.   Heliacal rising and setting of planets.

Mars rises heliacally in the East by 28°, Jupiter by 14°, Saturn by 17° of S'īghra anomaly and set heliacally in the west by degrees which are the differences of the above and 360° respectively.

Comm.   The Sun's velocity being greater than that of the Superior planets, the Sun overtakes them so that they set in west and rise in the East.   When these planets are

situated within particular limits from the Sun, they will be invisible in the rays of the Sun. As these superior planets will be near the Sun, near the moment of conjunction, they will not be seen at conjunction and within particular limits from the position of the Sun. The limits cited above mostly depend upon their respective brilliance and to some extent upon their distances from the Sun too. Their brilliance again depends upon the extent of their gibbosity. The formula given for the phase in modern astronomy is phase $= \dfrac{1+\cos E p S}{2}$ and noting that $\widehat{Eps} = E_2 =$ the S'ighraphala, phase $= \dfrac{1+\cos E_2}{2}$. Hence the Superior planets are always gibbous ie. the disc illuminated will be always greater than $\frac{1}{2}$. Though at conjunction $E_2 = 0$ and the entire discs of the major planets will be illuminated, we cannot see them as they are immersed in the rays of the Sun. As they emerge out of conjunction gradually E_2 will be increasing so that the discs will be illuminated lesser and lesser gradually. But K decreasing, the brilliance will not be so much effected. Along the arc acb (figs. 11, 12) the planets gradually gain in illumination and will be brightest when they are in opposition both as cos E increases and K decreases. In other words the Superior planets appear more and more brilliant when they are retrograding, being most brilliant at 'C'. The spherical radii of Jupiter Saturn and Mars being in decreasing order, their brilliance will be in decreasing order so that they will be rising at distances from the Sun which are in increasing order. The inverse square law of courses works here but combining the two factors (1) the spherical radius of the planet and (2) the inverse square law, we may take it that Bhāskara's numbers cited above namely 28°, 17° and 14° for Mars, Saturn and Jupiter respectively accord with truth, for, these numbers are given by Bhāskara or his authority Brahmagupta only after observing the planets rising heliacally.

That the positions of the planets while setting or rising heliacally will be situated symmetrically with respect to the Sun, goes without saying.

But one thing. In the chapter called Udayāstāmayā-dhyāya, the degrees known as Kālāmsas which are given as the arcs between the planets and the Sun for heliacal rising or setting, are different from the numbers given above, for, the latter are the values of the Sīghra anomaly. In the case of Mars when the Sīghra anomaly is 28°, the Sīghraphala will be 11°, so that the apparent distance of the planet will be 28°−11°=17° which are the Kālāmsas for Mars. Similarly in the case of Jupiter, when the Sīghra anomaly is 14°, the Sīghraphala will be 3°, so that the distance between the planet and the Sun as seen from the earth will be 11°, which are given as Jupiter's Kāla-msas. Also in the case of Saturn, the Sīghraphala for 17° of anomaly will be 2°, so that the Kālāmsas would be 15°.

Verse 43. Mercury and Venus rise in the West by 50° and 24° of Sīghra anomaly respectively, and set in the West by 155°, and 177° respectively. They rise in the East by 205° and 183° of Sīghra anomaly and set there by 310° and 336° respectively.

Comm. Regarding the Inferior planets, they rise heliacally in the East after Inferior conjunction and then they are retrograde. They attain gradually the maximum elongation in the East and after they revert to direct motion, their elongation gradually decreases. They then set in the East and heliacally rise thereafter in the West. There again their elongation attains a maximum value; then they begin to retrograde and gradually set in the West only to rise in the East. This is all clear from the heliocentric figure 11. When the Sīghra anomalies of Mercury and Venus happen to be respectively 50° and 24°, their Sīghraphalas would be 13° and 11° respectively, so that they are themselves the Kālāmsas, in as much as in

the case of the Inferior planets, the S'īghraphala will be itself their elongation eastern or western. Then they rise in the West, being near Superior conjunction. When again their S'īghra anomalies equal respectively 155° and 177°, the same S'īghraphalas will arise so that they set heliacally in the West. Then as the S'īghra anomalies attain the symmetrical values on the other side ie. (360–155) and (360–177) ie. 205° and 183° the S'īghraphalas being the same, they rise in the East. Again when they attain the values (360–50) and (360–24) ie. 310° and 336°, they set in the East on account of the same S'īghraphalas or Kālamsas.

Verse 44. When the S'īghra anomalies have particular values, to decide when the planets rise or set heliacally, we have to take the difference of those particular values and the numbers given above for the respective S'īghra anomalies, convert them into minutes of arc and divide the results by the daily motion in the S'īghra anomalies in minutes of arc. Then we have the number of days in which the rising or setting takes place thereafter.

Comm. Suppose it is required when Mercury rises in the West. Suppose we want to compute this on a particular day when Mercury's S'īghra anomaly is x°. Then because we know that Mercury rises in the West when his S'īghra anomaly is 50, we have to calculate by how many days the difference $| x - 50 |$ of the S'īghra anomaly would be covered. Let the daily motion of the S'īghra anomaly be y' per day, Then by rule of three $\dfrac{| x - 50 | \times 60}{y}$ will be the number of days before or after as the case may be for Mercury to rise in the West.

Verse 45. To obtain the Mean planet knowing the True.

Assume the True planet to be the Mean; compute the Manda and S'īghraphalas and applying them inversely,

we have an approximation of the Mean planets. Treating these as the Mean planets, again obtaining the Manda and Sīghraphalas and again applying them inversely and repeating the process till constant values are obtained, we have by this method of successive approximation the Mean planets required.

Comm. The method of successive approximation is clear.

Verse 46. To obtain the equinoctial shadow.

Convert the Ayanāmsas into minutes of arc, and divide by the mean daily motion of the Sun; then we have the number of days before the Meṣa or Tulā Samkrānti day, or before Makara and Karkataka Samkrānts days, when the Sun will be in equinoxes or Solstices respectively. The mid-day shadow of the Sun cast by the gnomon on such an equinoctial day, will give us the equinoctial shadow required.
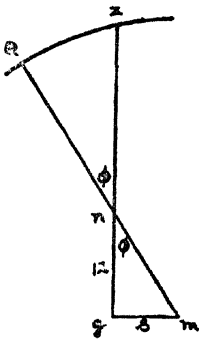
Fig. 18

Comm. The palabhā or equinoctial shadow as it is called is the length of the shadow cast by a gnomon taken to be of 12 units in length, (measuring the shadow also in the same units), at noon of an equinoctial day. In other words, if this shadow be of s units, clearly $\frac{s}{12} = \tan \phi$ (Vide fig. 18).

The Ayanāmsas are the degrees of the arc of the ecliptic in between the Hindu Zero point of the Zodiac and the first point of Aries which is now behind the former due to the phenomenon known as the precession of the equinoxes. They are called Ayanāmsas because the solstices are also behind the Makara and Karkataka Samkrānti points of the Hindu Zodiac or points which have Hindu longitudes 270° and 90° resply, by the same arc. The Hindu astro-

nomers came to know that the solstices are preceding by observations made with the gnomonic shadow at mid-day around the solstitial days. The day on which the maximum mid-day shadow is cast by the gnomon is the Winter solstitial day whereas the day on which the mid-day gnomonic shadow is maximum in the southern direction (assuming the place to be of northern latitude and $\phi < \omega$) or minimum in the northern direction ($\phi > \omega$) is the summer solstitial day. Calculating the Sun's longitude on that day at noon, we know how far the solstitial points have preceded behind the points of the Hindu Zodiac which have Hindu longitudes 90° and 270°. The word अयनांशाः has therefore the meaning अयनविलोमगत्यंशाः where the Samāsa may be viewed as a

Verses 47, 48. Calculating the five fundamental H sines of a point of the Zodiac pertaining to a point of the ecliptic.

The declination has to be computed from the sum of the Hindu longitude of that point and the Ayanamsas. Similarly if it be required to find the time before or after the rise of a point of the ecliptic, we have to compute them from the sum of the longitude of that point and the Ayanamsas.

$$\frac{\text{H sin } 24° \times \text{H sin } \lambda}{\text{R}} = \text{I}$$

$$\text{H sin}^2 \delta = \text{H cos } \delta = \text{Dyujyā} \qquad \text{II}$$

$$\delta = \text{H sin}^{-1} (\text{H sin } \delta) \qquad \text{III}$$

$$\frac{s}{12} \times \text{H sin } \delta = \text{Kujyā} \qquad \text{IV}$$

$$\frac{\times \text{R}}{\text{H cos } \delta}$$

$$\text{H sin}^{-1} (\text{Charajyā}) = \text{Charam} \qquad \text{VI}$$

Comm. (1) Let rA⊙ be the ecliptic where $r =$ Vernal Equinox, A = first point of the Hindu Zodiac,

in between r and $\odot$, not shown in the figure $\odot$ = the Sun. Let rA = a° = Ayanamsas defined before so that $r \odot = (a + \lambda)$. From the spherical triangle $r \odot$M, by Napier's rule, sin δ = (sin λ + a) sin ω which in Hindu trigonometry becomes H sin δ =

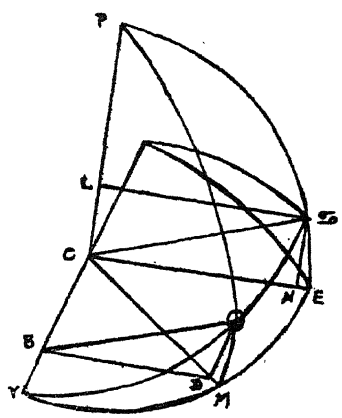$$\frac{H \sin (\lambda + a) \times \sin \omega}{R}.$$

Fig. 19

In Hindu Astronomy the obliquity of the ecliptic was taken to be 24°. The value of the obliquity is now 23°-27′ approximately and it has been know that this has been decreasing. At the time when the Hindu Astronomers observed this, it should have been greater than 23°-27′ so that if it was taken to be 24°, their observations were not far from truth. This means that the antiquity of Hindu Astronomy might be far more than what the Moderns estimate it to be.

(2) How the formula I was derived in Hindu trigonometry was as follows. Let $\leftharpoonup$ be the summer solstice $\leftharpoonup$C, $\odot$B, the perpendiculars dropped from $\leftharpoonup$ and $\odot$ on the line of intersection of the Equatorial and Ecliptic planes namely rBC. Let perpendiculars be dropped from $\leftharpoonup$ and $\odot$ on the plane of the Equator. Let them be $\leftharpoonup$N, $\odot$D, Join NC and DB. Then $\odot \widehat{BD} = \leftharpoonup \widehat{CN}$ = the dihedral angle between the two planes = ω; C$\leftharpoonup$ = R since in Hindu trigonometry H sin 90° = R. Also $\leftharpoonup$N = H sin $\leftharpoonup$E (in Hindu trigonometry) = H sin ω; $\odot$D = H sin $\odot$M = H sin δ; $\odot$B = H sin $r \odot$ = H sin (λ + a) where λ is the Hindu longitude of the Sun and a = Ayanamṣas. It will be seen that $\odot$D is a segment of the

line of intersection of the planes PCM a plane perpendicular to the plane of the Equator, and ⊙BD a plane perpendicular to the Ecliptic plane. Since ⌣C 11 ⊙B and ⌣N 11 ⊙D and BD⊙ = CN⌣ = 90°, so the two triangles are congruent

$$\therefore \frac{\odot B}{\smile C} = \frac{\odot D}{\smile N} \text{ ie. } H \sin (\lambda + a) = \frac{R \times H \sin \delta}{H \sin \omega}$$

$$\therefore H \sin \delta = \frac{H \sin (\lambda + a) \; H \sin \omega}{R}$$

$$NC = \smile L = \sqrt{\overline{C\smile^2} - \overline{\smile N^2}} = \sqrt{R^2 - H \sin^2 \delta} =$$

H cos δ. ⌣L is called Dyujyā as explained below in note (3).

Another way of looking at the similarity of the triangles C⌣N and B⊙D is from the fact that they are formed by the intersection of parallel planes, both of which are perpendicular to the plane of the Equator. This idea will be elaborated when we explain fig. 21, wherein the so-called latitudinal triangles will be shown to be formed by the intersection of the plane of the Equator and planes of diurnal circles with the planes of the horizon and the prime vertical. In fact, the plane of a great circle and the parallel planes of the corresponding small circles form the same dihedral angle with the planes of the celestial sphere namely the planes of the meridian, horizon and prime-vertical so that right-angled triangles formed by their intersection will be all similar.

Formula II is derived from the formula $H \sin^2 \delta + H \cos^2 \delta = R^2$ derived from fig. 6 from which formula, formula III follows.

(3) Why H cos δ is called Dyujyā in formula II is clear from fig. 20 wherein p is the celestial pole, (C) is the

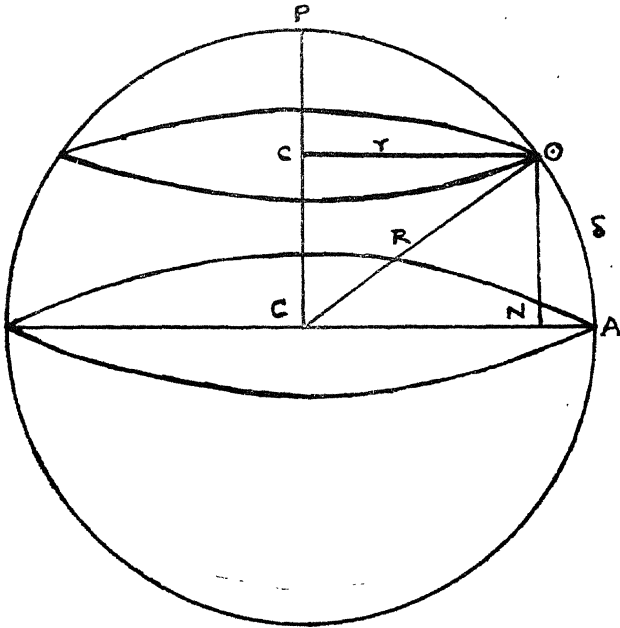**Fig. 20**

celestial Equator and (C) is the diurnal circle of a celestial body say the ⊙ ie. the Sun. Let ⊙N be the perpendicular dropped from ⊙ on the plane of the Equator so that ⊙N = H sin ⊙A = H sin δ CN = H cos δ = C⊙ = radius of the diurnal circle called Dyujyā (Dyu = day) द्युवृत्तज्रिव्या = द्युज्या (Madhyamapadalōpi Samāsa) formulae IV, V and VI will be dealt with in the next chapter Tripraśnādhyāya more elaborately but one has to understand what Charajyā is to follow the subsequent verses of this chapter so that we shall give its location and definition in the light of fig. 21. Let fig. 21 represent the celestial sphere in which SEN = Horizon, Z = Zenith, N = Nadir Ez = prime vertical EQR = celestial equator,
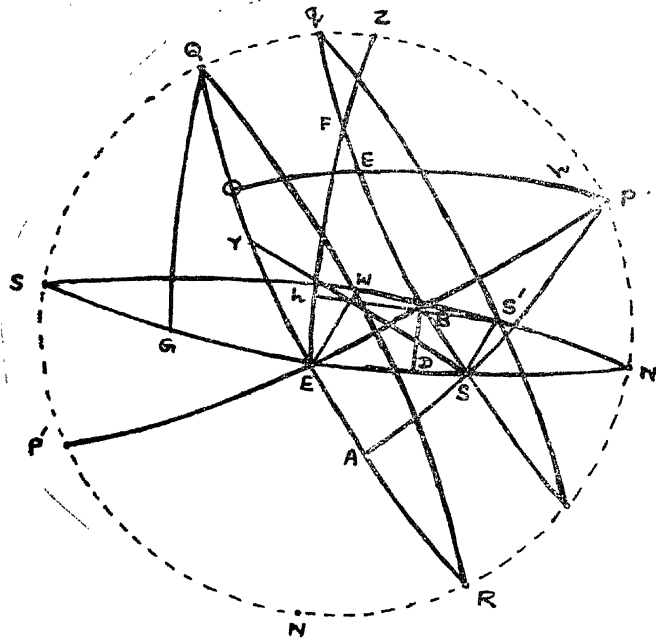
Fig. 21

PP' = Polar axis, SBS' = the diurnal circle of a celestial
body S; EW = the East-West line, SS' = Udayāstasūtra
or the join of the rising and setting points S, S'. This
SS' is evidently a diameter of the diurnal circle which is
bisected by the plane of the horizon; PEP' is called the
unmandala or the Equatorial horizon. PSA is the decli-
nation circle of S cutting the Equator in A. EA is called
the charam whose H sine is called Charajyā. The H sine
of SB in the diurnal circle is called Kujyā so that as
corresponding lines in the diurnal circle and the Equator
stand in the ratio H cos δ : R,

$$\frac{\text{Kujya}}{\text{Charajyā}} \qquad \frac{\text{H cos}}{\text{R}} \qquad \qquad \text{I}$$

In the beginning of the Tripraśnādhyāya, Bhāskara says
" साक्षे देशे खगोलवलयानां, तिरश्चांनमगोलवलयानां च संपातात्र्यस्राणि
क्षेत्राण्युरःचन्ते, तान्यक्षक्षेत्रसंज्ञां ie. In a place having a
latitude the diurnal paths of stars and planets will be
inclined to the fundamental circles of the celestial sphere
namely horizon, meridian and prime vertical and so their
intersection gives rise to what are called latitudinal tri-
angles, in which the angles would be ϕ, 90-ϕ and 90
where ϕ is the latitude. Thus in fig. 21 the projections
of the triangles ESB, EDB, DSB, EDF, EBF, ESF and
EQG on the meridian plane will be all triangles in which
the angles will be ϕ, 90-ϕ and 90° so that they are all
latitudinal triangles. These are all similar to the funda-
mental gnomonic triangle gmn of fig. 18 wherein also the
angles are ϕ, 90-ϕ and 90°. Here, there is one important
point to be observed. We have said "their projections
are all similar". In fact the corresponding spherical
triangles enumerated above are all apparently similar,
though they are not viewed in modern astronomy as
regular spherical triangles, because all the three sides of
the triangles are not arcs of great circles. It is not possi-
ble to apply Napier's rules to these triangles for the reason
mentioned above. None the less, the property of simil-
arity of the projected right angled triangles is made use
of in Hindu Astronomy to obtain the magnitudes of the
sides of the projected triangles.

EW is called the prāk-pratichī sūtra, SS' the Uda-
yāsta sūtra; similarly if lines through F, B, D, A parallel
to EW be drawn, these sūtras will intersect the meridian
planes in points which constitute the projected triangles
mentioned above. Let us study these triangles which we
connote by the same letters with lowered indices. Thus
for example $E_1 S_1 B_1$ is the projected triangle of ESB
where of course E_1 will be the centre of the celestial
sphere. The lines drawn parallel to SS' or EW through
B, D etc. will be denoted as BB', DD', FF' etc.

In the triangle $E_1 S_1 B_1$, $E_1 S_1$ is called Agrajyā, $S_1 B_1$ Kujyā, and $E_1 B_1$ Krāntijyā. Of the sides of ESB. ES and EB are arcs of great circles whereas SB is the arc of a small circle. In the projected triangle $E_1 S_1 B_1$, $E_1 S_1$ will be equal to the perpendicular from S on Eω, which will be H sin (ES) and is called Agrajyā ; $E_1 B_1$ will be equal to the perpendicular from B on Eω which will be H sin BE and as such called Krāntijyā. But $S_1 B_1$ which is equal to the perpendicular from S_1 on BB' the diameter of the diurnal circle is the H sine of SB in the diurnal circle and is called Kujyā. It will be noted that the perpendiculars from S on Eω, and BB' and the perpendicular from B on Eω do not form a triangle by themselves but by the theorem of three perpendiculars, if SL be the perpendicular from S on the plane of the unmandala ie. the great circle EBP, and if LM be perpendicular from L on Eω, SM will be perpendicular on Eω. Here the perpendicular SL will be the Kujyā, and SM the Agrajyā, whereas LM is not actually the H sine of BE but is equal and parallel to it.

Similarly take the spherical triangle SAE. This is a regular spherical triangle because the three sides are arcs of great circles. Napier's rules can be applied to this triangle and we have the formula sin SA = sin SE sin $\widehat{\text{SEA}}$ or in Hindu form H sin SA = $\dfrac{\text{H sin SE} \times \text{H sin } \widehat{\text{E}}}{\text{R}}$ or RH sin δ = Agrajya $\times$ H cos ϕ ie. Agrajyā =

$$\frac{\text{RH sin } \delta}{\text{H cos } \phi} \qquad \text{II}$$

Again sin EA = $\underline{\text{tan SA}} \times \text{tan } \widehat{\text{E}}$ where H sin EA is called Charajyā and tan E = cot ϕ so that Charajyā = tan δ tan ϕ in modern form, and the Hindu form is R tan δ tan ϕ

From I $\dfrac{H \cos \delta}{R} = \dfrac{Kujyā}{Charajyā}$ so that

$$Kujyā = \dfrac{H \cos \delta}{R} \times R \underline{\tan \delta} \tan \phi = \underline{H \sin \delta} \tan \phi \quad \text{III}$$

In Tripras'nādhyāya, the elements of all the eight latitudinal triangles are found by using their similarity with the fundamental gnomonic triangle. The important elements that will enter into computation are (a) Charajyā (b) Kujyā (c) Agrajyā (d) Taddhriti ie. $S_1 F_1$ the projection of SF on the plane of the meridian (e) Sama-S'anku $= E_1 F_1 = H \sin EF$ (f) Krāntijyā $= E_1 B_1 = H \sin EB$ $= H \sin \delta$ (g) Lambajyā $= H \sin QS = H \cos ZQ =$ $H \cos \phi$ (h) Akshajya $= H \sin ZQ = H \sin \phi$ (i) $B_1 D_1$ $= Ud\text{-}Vritha\text{-}S'anku = H \cos ZB$ (j) Dinārdha-S'anku $=$ $H \cos Zq$. The following points will be noted.

(i) In the triangle $E_1 Q G_1$, the projected triangle of EQG on the meridian plane, noting that E_1 is the centre of the celestial sphere $E_1 Q = R$, $Q G_1 = H \cos \phi$ and $E_1 G_1 = H \sin ZQ = H \sin \phi$.

(ii) $S_1 f_1 = H \sin SB + H \sin fB$ (both the H sines pertaining to the diurnal circle.

(iii) Sama S'anku is the H cosine of the Zenith-distance when the Sun or celestial body is on the prime-vertical.

(iv) BD is an arc of the great circle ZB, so that the Unmandala S'anku is the H cosine of ZB.

(v) Dinārdha — S'anku $= H$ cosine ZQ.

(vi) These S'ankus are the H sines of altitudes or H cosines of Zenith-distances and they are in the planes of the respective great circles.

(vii) Lambāmsa-chāpa is the arc of the colatitude
QS where S is the South point so that Lam-
bajyā is the H sine of QS or H cosine of ZQ.
This Lambajyā will be seen to be the diameter
of a small circle parallel to the Equator and
passing through Z.

Verses 49–51. A different method of obtaining
chara.

This chara can be had by the so-called chara-segments
of the locality using a process similar to that of finding
the H sines of the smaller table of nine H sines using
$\frac{\lambda}{3}$ where λ is the Sāyana longitude of the Sun (i.e. The
Hindu longitude plus the arc of ayanāmsʹas is called the
Sāyana longitude or the modern longitude measured from
ν along the ecliptic to the Sun). Find the charas of 30°,
60° and 90° of the ecliptic measured from ν. Subtract the
first from the second, the second from the third. Thus we
have C30°, C60°–C30°, C90°–C60°, (where Cθ° signifies the
chara of θ°) which are called chara-Khandas or chara-seg-
ments. The equinoctial shadow multiplied by 10, 8, 3⅓
gives the approximate values of the chara-segments in
Vinādis (a sidereal day is divided into 60 nādis and each
nādi consists of 60 Vinādis). The chara–segments thus
measured in Vinādis are rather approximate. If further
exactitude is required, better take the arc in units each
of which rises in ⅙th of a Vinādi (This ⅙th part of a
Vinādi is known as a prāna i.e. the duration of the
interval between two inhales of a healthy person reckoned
as 4″ of time).

Comm. The charas of 30°, 60° and 90° of modern
longitude are the values of the arc EA, (Ref. Fig. 21)
when S has longitudes 30°, 60° and 90°. We have the
folmula

Charajyā $= R \tan \delta \tan \phi$

Taking the equinoctial shadow equal to 1 Angula means tan $\phi = \frac{1}{12}$. As charajyā is proportional to tan ϕ, for any equinoctial shadow of s angulas, tan ϕ being equal to $\frac{s}{12}$ the charajya got above is to be multiplied by S only to obtain the charajyā in any place where the equinoctial shadow is s angulas. Putting the modern longitudes equal to 30° & 60°, if the corresponding declinations be δ_1, δ_2 sin δ_1 = sin 30 sin ω, sin δ_2 = sin 60 sin ω. Taking $\omega = 24°$ and applying logarithmic tables

log sin δ_1 = 9.6990 + 9.6093 = 9.3083 so that δ_1=11°-44′

log sin δ_2 = 9.9375 + 9.6093 = 9.5468 so that δ_2=20°-38′

Now from the formula for charajya cited above viz. H sine (chara) = R tan δ tan ϕ or sine (chara) = tan ϕ tan δ, putting tan $\phi = \frac{1}{12}$ and applying tables, using the values of δ got above,

$$\text{charajya for } 30° = \frac{\tan 11°\text{-}44′}{12}$$

$$\text{and charajyā for } 60° = \frac{\tan 20′ - 38′}{12} \quad \text{so that}$$

log (sine chara) = 9.3175−1.0792   for 30° and for 60°

log (sine chara) = 9.5758−1.0792

∴   Chara for 30° or C (30)° = 59′ and C (60°) = 1° − 48′ Converting these arcs into their rising times at the rate of 6′ per Vinadi, we have C (30°) = 10, and C (60°) = 18

Noting δ_3=ω, sin (chara) for 90° = $\frac{\tan 24°}{12}$ so that

log sin (C 90°) = 9.6486 − 1.0792 = 8.5694   so that

C (90°) = 2°-8′ = $21\frac{1}{3}$ Vinādis.

Thus (C $\overline{30}$) = 10, C (60)−C (30) = 18 − 10 = 8 C (90°) − C (60°) = $21\frac{1}{3}$ − 18 = $3\frac{1}{3}$ so that the chara Segments are respectively 10, 8, $3\frac{1}{2}$ as given by Bhāskara.

For a given place of equinoctial shadow equal to s″, we have to multiply 10, 8, 3½, by s, which will be the chara-segments for the place.

The meaning of the first half of the verse 49 is as follows.—Let the equinoctial shadow for a place be 3 Angulas. Then the chara-segments for that place are 30, 24, 10. These are three in number for a longitude λ of 90°. If the longitude be 44° (say) then proceed as we have done to find sin 24°, using the method of Bhogya Khanda Sphutīkaraṇa with respect to the table of H sines namely 21, 20, 19, 17, 15, 12, 9, 5, 2. Proceeding as directed the Sphuta

Bhogya Khanda for 14° is $\left(\dfrac{3 \times 14}{30} = \dfrac{7}{5}; \; 30 - \dfrac{7}{5} = 28\frac{2}{5}\right.$

$\dfrac{14 \times 28\frac{2}{5}}{30} = \dfrac{2002}{150} = 13\frac{1}{3}; \; 30 + 13\frac{1}{3} =$

Proceeding according to the modern formula we have

Charajyā = tan δ tan ϕ where $\phi = \frac{3}{12}$, and sin δ = sin 44° sin 24°

log sin δ = 9.8418 + 9.6093 = 9.4511; δ = 16°-25

log tan δ=9.4693 ∴ log sin (chara)=9.4693+log tan ϕ

= 9.4693 + log $\frac{3}{12}$ = 9.4693 − .6021 = 8.8672

∴ Chara = 4°-14′ = $\dfrac{254}{6}$ = 42⅓ Vinadis whereas we have

got by the Hindu method 43⅓ which is near the truth.

Verse 52. To find the durations of day and night.

Fifteen ghatis increased or decreased by the Chara-nadis, according as the Sun is in the northern hemisphere or southern, gives half the day of the locality and the difference of 30 nādis and the above half-day gives half the duration of night.

ः ः *Comm.* Ref. fig. 21. Let S be the Sun rising in the hemisphere when his declination is north. Then

from the figure $\widehat{APQ}$ is the rising hour-angle $=\widehat{APE}+\widehat{EPQ} = \widehat{APE} + 90°$ where 90° correspond to 15 nādis. So we have to add $\widehat{APE}$ expressed in nādis equal to the rising time of AE. Let us find the duration of the day for the place where s = 3″ and when λ of s = 44°; the latitude of the place will be 14° (from tables) when λ 44°, we have found above that 42-20 Vinadis is the chara expressed in time. Hence half the day = 15-42-20 or duration of day 31-25; duration of night = 28-35.

Note (1) In modern astronomy we have the formula cos h = — tan ϕ tan δ where h is the rising hour angle. Putting h = H + 90° where H stands for the arc EA of fig. 21 cos (90+H) = — sin h = — tan ϕ tan δ so that sin H = tan ϕ tan δ ie. H sin (Chara) = R tan ϕ tan δ which accords with the formula found before. The word chara used for EA means etymologically रविसञ्चारवशेन दिन-विकार: i e. the variation in 15 ghatis of the eqiunoctial half-day on account of the Sun's variation in declination,

(2) If ϕ = 0, Chara = 0 so that duration of half-day is 15 ghatis ie. on the terrestrial equator, whatever be the Sun's declination, the length of the day will be always 12 hours.

(3) Let δ = 0 so that chara = 0 ie. whatever be the latitude (provided ϕ ≯ 90-δ as we shall see shortly). the day and night will be each of 12 hrs.

(4) Let δ be negative, so that the arc connoting chaaa ie. EA will be above the horizon, and consequently the duration of half-day will be less than 6 hrs. by the time that is taken for EA to rise. This can be seen otherwise also as cos h = — tan ϕ tan δ = + ve so that h<90° which means half-day is less than 6 hrs.

(5) We could also treat the case when ϕ is negative, but as this case is not in the purview of Hindu Astronomers who had only India in their mind and as such were concerned primarily with positive latitudes. If, however, we consider a negative latitude, when δ is + ve, cos h will be positive and if δ is — ve, cos h will be negative. This means that when the Sun is in northern latitudes, the southern latitudes will have their day less than 12 hours and when the Sun is in southern latitudes, their day will be greater than 12 hrs.
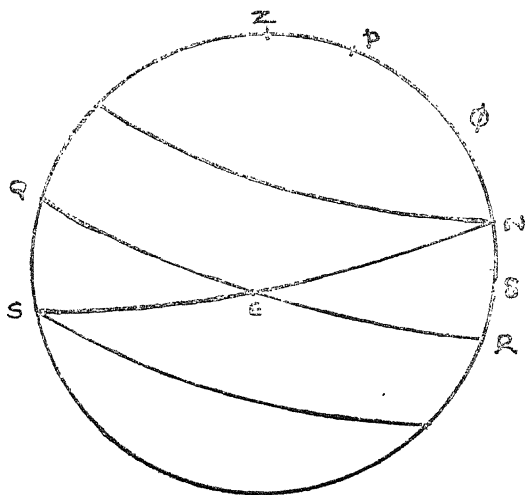
Fig. 22

(6) Let $\phi + \delta = 20°$. (Ref. fig. 22) ie. imagine
the Sun to rise at N, the north point so that RN + NP =
$\delta + \phi = 90°$. Then the Sun's diurnal path will be entirely
above the horizon, which means that what is called
'perpetual day' begins for that place on that day and lasts
as long as $\delta \geqslant 90 - \phi$. For the same place, let $\delta =$
$- (90 - \phi) = QS$. In this case the Sun sets at S, and
what is called perpetual night begins and lasts till the
southern declination of the Sun is greater than $90 - \phi$.
The duration of perpetual day can be found as follows
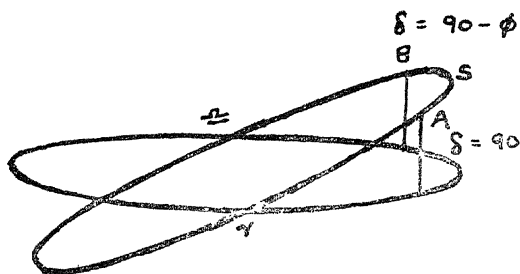which applies to the perpetual night as well.

Fig. 23

(Ref. fig. 23). Let A be the point at which the
declination is $90 - \phi$; let S be the summer solstice and let
B be the point where again the declination is equal to
$90 - \phi$. So long as the Sun traces the arc AB = 2 AS =

$(90-r\mathrm{A})$, there will be perpetual day. But $\sin \delta = \sin \lambda$ $\sin \omega$ so that $\sin \lambda$

$$= \frac{\sin \delta}{\sin \omega}. \quad \text{Putting } \delta = 90 - \phi, \; \sin \lambda = \frac{\cos \phi}{\sin \omega};$$

$$\text{Sin } \lambda = \sin \gamma\mathrm{A} = \cos \mathrm{AS} \quad \therefore \cos \mathrm{AS} = \frac{\cos \phi}{\sin \omega}$$

$$\therefore 2 \mathrm{AS} = 2 \cos^{-1}\left(\frac{\cos \phi}{\sin \omega}\right). \quad \text{Supposing AS expressed}$$

in degrees and assuming the Sun goes along the ecliptic with uniform motion, since he takes $365\frac{1}{4}$ days to trace $360°$, to trace 2 AS, he takes $\dfrac{2\,\mathrm{AS}}{360} \times 365\frac{1}{4}$ days $= \dfrac{365\frac{1}{4}}{180}$

$\times \cos^{-1}\left(\dfrac{\cos \phi}{\sin \omega}\right)$ which is the length of the perpetual day.

Verse 52. The correction known as Chara.

The daily motion of the planet being multiplied by the chara expressed in asus and divided by the asus in a day viz. 21659 and the result being subtracted from or added to the planetary position at Sunrise according as the Sun is in the northern or southern hemisphere. The result is to be added to or subtracted from the planetary position at Sunset.

Comm. The mean planets computed hold good at the Sun-rise at Lanka i.e. at zero latitude; they have to be converted to hold good at the local Sun-rise. In other words in fig. 21, the mean planet computed is B which is on the Lanka horizon, whereas we have to get S, the same planet on the local horizon. The position of S is earlier than that of B by the time interval indicated by the arc SB which is measured by EA the chara because the position B on the Lanka horizon is later than the position S on the local horizon. If the mean planet moves in a day of 21659 asus by the arc denoting its daily mean motion, by how much does it move in the time of chara expressed in asus? The result is

$$\frac{\text{Chara in Asus} \times \text{mean daily motion}}{21659}.$$

24

This result is to be subtracted from the position of B to get the position of S. If the position B′ which is the setting position at Lanka, and if we have to get S′ the local setting position, in as much S′ is later than B′ by the same arc, we have to add the above result to the position B′ to get S′. Here the asus in a day is given to be 21659 and not 60 × 60 × 6 = 21600 because the mean daily motion is during the course of the Sāvana day i.e. the true solar day and not during the course of a sidereal day of 21600 asus. The Sāvana day exceeds the sidereal by the time taken by the arc moved by the Sun during that day which is very approximately 59′. Since a minute of arc of the Equator rises in an asu i.e. in 4″, so 59′ of the Sun's motion is covered in 59 asus which is to be added to 21600 asus of the sidereal day to get the Sāvana day approximately. It will be noted that this correction of chara in the planetary positions is due to latitude of the place and if the latitude is zero, it need not be done. Also if $\delta = 0$, it need not be done, for, then, the Sun or the celestial body whose $\delta = 0$ will be rising at E itself in which case the chara EA will be zero.

Verses 54, 55. The H sines of 30°, 60° and 90°, being squared and decreased by the squares of their respective declinations, the square-root of the differences being taken, and the result being multiplied by the radius, is to be divided by the respective H cosines of the declination. The first of the three results, the difference of the second and the first and the difference of the third and the second will give us the rising times of what are called the Sāyana rasis of Mesha, Vrishabha and Mithuna; their reverses will then give the rising times of the next three; then the original ones those of the next three and again then reverses those of the last three.

Comm. There are twelve Rasis in the Zodiac of equal interval. Measuring from *r*, and taking arcs of the

ecliptic successively each of 30°, we have what are called
Sāyana Rasis or Rasis taking the Ayanāmśa into consi-
deration or in other words measuring from r. On the
other hand successive arcs each of 30° measured from the
zero point of the Hindu Zodiac, constitute what are called
the Nirayana Rāśis. The words Meṣa, Vriṣabha etc.
signifying the shapes of the constellations apply strictly
to the Nirayana Rāśis. But nonetheless, by the conven-
tion what is called Upachāra, we name the Sāyana Rāśis
also by the same names. In as much as the zero-points
of the Hindu Zodiacs is ahead of the modern zero-point by
an arc which is the arc of Ayanamsa or accumulated pre-
cession, the words Sāyana and Nirayana came into vogue.
Once upon a time approximately in Varāha's time or
rather 499 A. D, the two zero-points coincided and then
the Sāyana and Nirayana Rāśis were the same. Gradually
on account of the phenomenon of precession r preceded,
and today the distance between r and the Hindu zero-
point is about 20°–30′. Of late, there has been a big
controversy as to what exactly the arc is and the Calendar
reform committee has adopted a value far more than what
could be justified. The present author opines, that we
have no right to set aside a statement of no less an astro-
nomer than Varāhamihira who stated explicity ' आश्लेषार्घात्
दक्षिणमुत्तरमयनं रवेर्घनिष्ठाद्यम्, नूनं कदाचिदासीत् येनोक्तंपूर्वशास्त्रेषु,
साम्प्रतमयनं सवितुः कर्कटकाद्यं मृगादितश्चाऽन्यत् उक्तमाचो विकृतिः
प्रत्यक्षपरीक्षणव्यैकिः" i.e. True it is that once upon a time,
the Sun began his southern journey when he was mid-way
the constellation of Asleṣa and his northern when he was
at the beginning of Dhaniṣtha, because it was stated in
ancient texts. (The allusion is to Vedānga Jyotiṣa
where it was stated as such); but now the southern journey
of the Sun begins from the point marking ¾ of the con-
stellation of punarvasu which is the beginning point of
the Karkata Rāśi and his norther from the beginning of
Makara i.e. from the point marking ¼th of the constellation
of Uttaraṣādha ; there has been a change from what was

stated in the ancient texts; let people verify this by actual observation ". Accepting an Ayanāms'a which goes against the statement of this great astronomer, who said that he observed and called upon others to observe, is really unwarranted, especially when the adopted Ayanāms'a of the Calendar reform committee goes on the basis of surmises and consensus. We shall deal with this topic in further detail in an appendix to this work, because it is a really important issue and has been wrongly solved. The importance of knowing the exact value of the arc is clear when we observe that from the correctly computed planetary positions of modern astronomy this ayanāms'a is being subtracted to give the positions measured from the Zero-point of the Hindu Zodiac. An error in the Ayanāms'a therefore vitiates the positions obtained by the above method.

Coming to the point, the problem on hand is to obtain the rising times of the Sāyana Rās'is at a place of zero latitude i.e. what are called Lankōdaya times of Sāyana Rās'is (Ref. fig. 21). Let $rS = 30°$ so that rA the equatorial arc gives the rising time of rS. We could have the magnitude of this arc by the formula derived from Napier's rules namely $\cos \omega = \tan \alpha \tan \lambda$ I.
But as in Hindu trigonometry the tangent functions of angles are not used, Bhāskara gave the following formula

$$\text{Sin } \alpha = \frac{\sqrt{\sin^2 \lambda - \sin^2 \delta}}{\cos \delta} \quad \text{II.}$$ This formula could be derived easily from formula I and the formulae $\cos \lambda = \cos \alpha \cos \delta$ III and $\sin \delta = \sin \lambda \sin \omega$ IV; for multiplying the right-hand sides of I and III we have $\cos \omega$

$$\cos \lambda = \frac{\sin \alpha \cos \delta}{\tan \lambda} \text{ i.e. } \sin \lambda \cos \omega = \cos \delta \sin \alpha$$

$$\therefore \sin \alpha = \frac{\sin \lambda \cos \omega}{\cos \delta} \quad \text{V.} \quad \text{But from IV } \sin \omega =$$

$\dfrac{\sin \delta}{\sin \lambda}$ so that $\cos \omega = \sqrt{1 - \dfrac{\sin^2 \delta}{\sin^2 \lambda}} =$

$\dfrac{1}{\sin \lambda} \sqrt{\sin^2 \lambda - \sin^2 \delta}$. Substituting in V for $\cos \omega$

we have

$H \sin \alpha = R \dfrac{\sqrt{H \sin^2 \lambda - H \sin^2 \delta}}{H \cos \delta} \sin \alpha =$

$\sin \lambda \times \dfrac{1}{\sin \lambda} \dfrac{\sqrt{\sin^2 \lambda - \sin^2 \delta}}{\cos \delta}$ $- \sin^2$
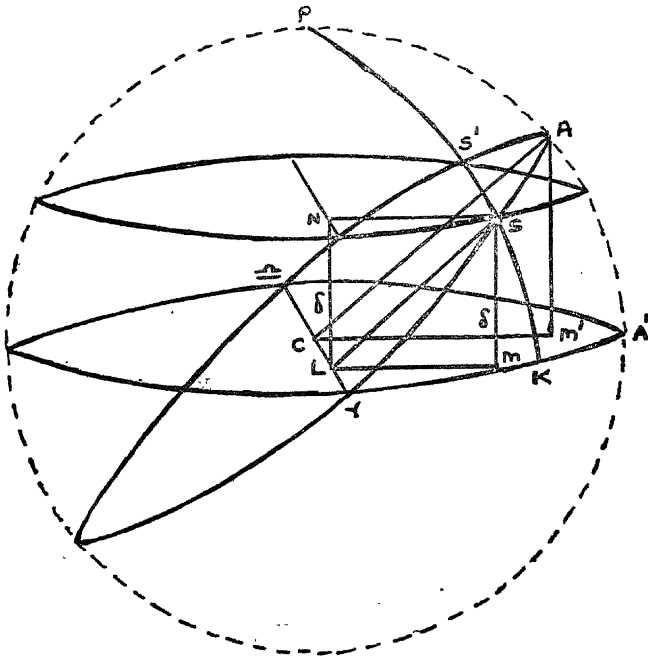
$H \cos \delta$ as stated. $\qquad \cos$

Fig. 24

But let us see how this formula was derived by the Hindu astronomers. (Ref. fig. 24). Let rS be an arc of the ecliptic equal to λ. Let SM be dropped perpendicular from S on the plane of the Equator so that $SM = H \sin \delta$.

From M drop a perpendicular ML on $r \simeq$ so that SL will be perpendicular from S on $r \simeq$ by the theorem of three perpendiculars. SL = H sin λ. Also ML will be equal to the perpendicular from S on the diameter of the diurnal circle of S parallel to $r \simeq$ so that ML = SN = H sine of the arc in the diurnal circle corresponding to Kr

$\therefore$ SW^2 = SL^2 — LN^2 = H sin^2 λ — H sin^2 δ

$\therefore$ SN = $\sqrt{\text{H sin}^2 \lambda \ - \ \text{H sin}^2 \delta}$

$\therefore$ The length of the perpendicular from K on $r \simeq$ =

R $\times \dfrac{\sqrt{\text{H sin}^2 \lambda - \text{H sin}^2 \delta}}{\text{H cos } \delta}$ since corresponding lines of

the diurnal circle and the equator stand in the ratio of H cos δ : R (Vide fig. 20). But this $\perp^{ar}$ is H sin α =

R $\dfrac{\sqrt{\text{H sin}^2 \gamma - \text{H sin}^2 \delta}}{\text{H cos } \delta}$.

If α_1, α_2, α_3 be the Right ascensions of the points on the ecliptic whose modern longitudes are 30°, 60° and 90°, expressed in asus, then α_1, α_2 — α_1, α_3 — α_2 will give the rising times of the arcs of the ecliptic which stand for Sāyana Meṣa, Sāyana Vriṣabha and Sāyana Mithuna. The rising times of the next three Rasis will be the same in reverse order since Karkata is symmetric with Mithuna with respect to the Equator and similarly Simha and Kanya symmetric with Vriṣabha and Meṣa. The next three are again symmetric with Meṣa, Vriṣabha and Mithuna and the last three with Mithuna, Vriṣabha and Meṣa.

Verse 56. The H cosines of the ends of the Rasis Karkata etc., being multiplied by the radius, and divided by the H cosines of their respective declinations, and the arcs of those H cosines being taken, subtract as before the preceding from the succeeding. Then we have the rising times of the Rasis beginning with Karkataka.

Comm. This is clear from fig. 24. If S be the end of Vriṣabha, S′ in the figure denotes the end of Karkata them from the right-angled triangle SAP,

$$\sin S'A = \sin SA = \sin \widehat{APS} \times \sin PS = \sin A'K \times \cos SK$$

$$\therefore \sin A'K = \frac{\sin S'A}{\cos SK}. \quad \text{But } \sin S'A = \cos S' \doteq$$

and $\cos SK = \cos \delta$ where δ is the declination at S.

A′K converted into time gives the rising time of AS′ ie. Karkata.

$$\therefore \text{ the rising time of Karkata} = \frac{\cos S'}{\cos \delta}$$

In the Hindu form, it will be

$$\text{Karkata-Rāsi-Udayakāla} = \frac{\text{Karkata-anta-Kotijyā}}{\text{Karkata-anta-Dyujyā}} \times R$$

as stated.

In modern terms $\sin A'K = \cos rK = \frac{\cos rS}{\cos \delta}$

from the formula $\cos \lambda = \cos \alpha \cos \delta$. Thus, virtually the formula is a statement of the formula $\cos \lambda = \cos \alpha \cos \delta$. Aslo the formula $\sin AS' = \sin \widehat{P} \sin PS$ is parallel to the formula $\sin \delta = \sin \lambda \sin \omega$ which we proved already from Hindu methods.

Verse 57. Still an alternative method.

The H sines of Meṣa etc. being multiplied by H cos ω divided by their respective H cos δ's and the arcs thereof being subtracted as before the preceding from the succeeding we have the rising times of Mesha etc.

Comm. From figure 24,

$$SN = SL \cos LSN \text{ and } \frac{SL}{\cos SK} = \text{perpendicular from}$$

K on

$\therefore \quad \dfrac{\text{SL cos }\omega}{\cos \delta} = \text{H sin }r\text{K}, \quad \text{SL} = \text{H sin }r\text{S}$

$\therefore \quad \text{H sin }r\text{K} = \dfrac{\text{H sin }r\text{S} \times \text{H cos }\omega}{\text{H cos }\delta}$

We proved the above in a modern way. The Hindu concept is derived from the similarity of SML and ACM′ where C is the centre of the sphere and M′ is the foot of the perpendicular from A on the plane of the equator

$\therefore \quad \dfrac{\text{LM}}{\text{CM}'} = \dfrac{\text{SL}}{\text{CA}} \qquad \therefore \quad \text{LM} = \dfrac{\text{H cos }\omega \times \text{H sin }r\text{S}}{\text{R}}$

Since LM = SN.

LM divided by H cos δ and multiplied by R gives H sin rK

$\therefore \quad \text{H sin }r\text{K} = \dfrac{\text{H cos }\omega \times \text{H sin }r\text{S}}{\text{R}} \times \dfrac{\text{R}}{\text{H cos }\delta} -$

$\dfrac{\text{H cos }r\text{S} \times \text{H cos }\omega}{\text{H cos }\delta}$

i.e. $\text{H sin }\alpha = \dfrac{\text{H sin }\lambda \times \text{H cos }\omega}{\text{H}}$

Here H cos ω is called Trigṛha-dyu-maurvī because it is the H cosine of the declination of λ when λ = 90°.

Verses 58, 59. The magnitudes of the rising times.

Those rising times are 1670, 1793, 1937 ; these in the same and reverse orders diminished or increased by their respective Chara segments which are also in the same and reverse orders give the rising times of the Sāyana Rasis beginning from Meṣa for the locality. The Rasis from Tulā are in a reverse direction i.e. as the Meṣa is projecting upwards above the horizon, Tulā will be projecting below the horizon so that, the time taken by Meṣa to rise is exactly the time taken by Tulā to set.

Comm. We shall compute the rising times of Sāyana Rasis for Lanka first i.e. for zero latitude using modern methods from the formula tan α = cos ω tan λ

log tan α=log cos ω+log tan λ ; Put λ_1=30, and λ_2 = 60 and take ω = 24° ; Let the corresponding α's be α_1, α_2

log tan α_1 = log cos 24 + log tan 30°   (1)

log tan α_2 = log cos 24 + log tan 60°   (2)

log tan α_1 = 9.9607 + 9.7614 = 9.7221

$$\therefore \quad \alpha_1 = 27° - 48'$$

log tan α_2 = 9.9607 + 10.2386 = 10.1993

$$\therefore \quad \alpha_2 = 57° - 42'$$

At the rate of 1 asu for 1', α_1 = 1668 asus α_2 = 3462 ; $\alpha_2 - \alpha_1$ = 1794 and since α_3 = the right ascension of 90° Longitude = 90°, α_3 = 5400   so   that $\alpha_3 - \alpha_2$ = 1938. These are given by Bhāskara as 1670, 1793, 1937, the first exceeding by 2 asus, the second and third each less by one asu and the total according with the total.   The rising times of Karkata etc. will be 1937, 1793, 1670, 1670, 1793, 1937, 1937, 1793, 1670 respectively.

Let us then find the rising times of these Sāyana Rasis at a locality say of latitude 13°.   Refer to fig. 21. Let rS represent Meṣa so that the rising times of rE is equal to that of rS.   But rE = rA — AE.   We have seen rA = 1670 using Bhāskara's value. Sin EA = tan 13° tan δ_1 where δ_1 is the declination of S where rS = 30°. Sin δ_1 = sin 30° sin 24°.

log sin δ_1 = 9.6990 + 9.6093 = 9.3083

$$\therefore \quad \delta_1 = 11° - 44'$$

$\therefore$ log sin EA = log tan 13° + log tan 11° — 44 =
$$9·3634 + 9.3175 = 8.6809$$

$\therefore$ EA = 2° — 45' $\therefore$ rE = 1670 — 165 = *1505* asus.

Similarly for λ = 60°, putting α_2, δ_2 in the place of α_1, δ_1 and proceeding as before rE = rA$_1$ — A$_1$E ; rA$_1$ = 3463; sin EA$_1$ = tan 13° tan δ_2 sin δ_2 = sin 60° sin 24°

$\therefore \quad \log \sin \delta_2 = 9.6093 + 9.9375 = 9.5468$

$\therefore \quad \delta_2 = 20° - 30'$

$\therefore \quad \log \sin \text{EA}_1 = 9.3634 + 9.5758 = 8.9392$

$\therefore \quad \text{EA}_1 = 4° - 59' = 299'$

$\therefore \quad r\text{E} = 3463 - 299 = 3164 \text{ asus}$

$\therefore$ Rising time of Sāyana Vriṣabha for the locality $= 3164 - 1505 = 1659 \text{ asus.}$

$r\text{A}_2 = 5400, \sin \text{EA}_2 = \tan 13 \tan \delta_2 = \tan 13 \tan 24°.$

$\therefore \quad \log \sin \text{EA}_2 = 9.3634 + 9.6486 = 9.0120$

$\therefore \quad \text{EA}_2 = 5° - 54' = 354 \text{ asus}$

$\therefore \quad r\text{E} = 5400 - 354 = 5046$

$\therefore$ Rising time of Mithuna is $5046 - 3164 = 1882 \text{ asus.}$

Before we proceed to find the rising times of Karkataka, Simha and Kanyā, we shall cast our previous procedure into the Hindu form. In fig. 21, let rS be the Sāyana Mesha. The rising time of rS is measured by rE, because when r is at E, Meṣa is just about to rise and when r is in the position indicated, the extremity of Meṣa namely S is rising. So it means that as rS of the ecliptic has risen, a portion rE of the Equator has risen. As time is measured by the arc of the equator which rises with a uniform speed, we measure the rising time of rS by the arc rE; but $r\text{E} = r\text{A} - \text{AE}$. rA is the Equatorial rising time of rS, because when A is at E, S will be at B i.e. A and S will then be on the equatorial horizon EB simultaneously. Hence $r\text{A} = 1670$ as proved before and stated by Bhāskara. EA is the chara for 30°. The chara for one angula or inch (inch is here used technically, and does not mean what it means in ordinary parlour) as has been stated by Bhāskara and proved by us is 10 Vinadis. (Vide page 181)

But we have taken 13° as our latitude, so that tan ϕ = .2309. Since $\frac{s}{12}$ = tan ϕ = .2309 $\therefore$ s = 2.7708; let us take this as 2.8″ so that the charas 10, 8, 3⅓ found for one inch are to be multiplied by 2.8 to give the local charas. They are in Vinādis, 28, 22.4, 9⅓ or in asus 168, 134·4, 56; for convenience let us take 134.4 as 134, so that the charas are 168, 134, 56. Thus the rising time of Sāyana Meṣa at this locality is 1670 − 168 = 1502 asus i.e. 250 Vinādis = 4-10 Nādis. Then let rS now represent 60°, instead of subtracting EA from rA to get the combined rising time of Meṣa and Vriṣabha, the Hindu practice is to subtract the chara pertaining to Vriṣabha from the equatorial rising of Vrishabha i.e. rE = 1793 − 134 = 1659 as got before. Here it must be noted that EA° is the chara not pertaining to Vriṣabha alone but to Meṣa and Vriṣabha put together. That is why for ease, the chara to Meṣa, the increase in chara for Vriṣabha, and the increase in chara for Mithuna as well as their individual equatorial rising times are given. The increments in the charas are called chara-khandas just as the increments in the H sines are called Jyā-khandas (khands means segments). Similarly the rising time of Sāyana Mithuna is equal to 1937 − 56 = 1881 asus = 313.5 Vinadis = 5-14 Nādis. Now with respect to Karkataka, its equatorial rising time is 1793, for, from fig. 25, the equator at the equatorial place being prime Vertical, if rM be Meṣa, its time of rising is given by rE where E is the foot of the declination circle of M. Similarly if MV represents Vriṣabha, when V comes to the horizon, N the foot of the declination circle comes to the horizon. Thus the rising time of any arc of the ecliptic at an equatorial place is given by the corresponding arc of the equator, which is intercepted between the declination circles of the ends of the arc. So from fig. 26 if r ed ≏ be the equator, rED. ≏ the ecliptic, A the Ayana or Summer solstice, P the pole rE, ED, DA etc. the Sāyana Rasis Meṣa etc. a, b, c etc. the feet of the
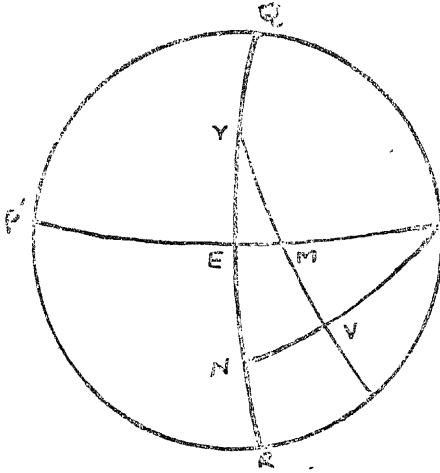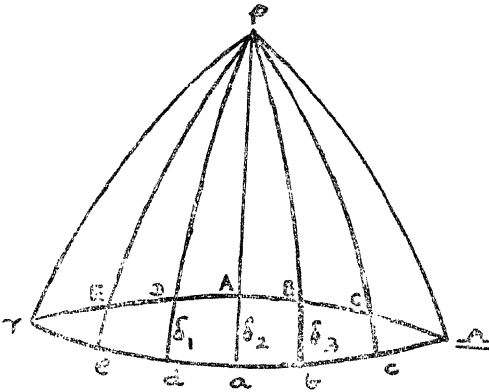
Fig. 25

Fig. 26

declination circles of A, B, C etc., since PAE is secondary
both to the ecliptic and equator (i.e. perpendicular circle)
spherical triangles PAD, PAB are congruent; PBC, PDE
are congruent and PC $\approx$ is congruent with PE r.   Hence
ab = ad i.e. rising times of Karkataka and Mithuna are
equal ; bc = de i.e. those of Simha and Vrisabha are

equal and similarly those of Meṣa and Kanyā. It will be noted that the equatorial risings alone are equal in the above cases but not at any other place, for in a place with some latitude when a point like B (fig. 26) comes to the horizon, the foot of its declination circle namely b will not be on the horizon and there arises the chara in between, which has to be taken into account, and be subtracted from or added to the equatorial rising time as the case may be.

Now regarding the chara-khanda of Karkata it is again 56; why it should be so is not proved by Bhāskara but merely stated, nor any commentator took the pains to prove. It can be proved as follows. Let in fig. 26, δ_1, δ_2, δ_3 be the declinations at the ends of Vriṣabha, Mithuna and Karkataka respectively. We know $\delta_1 = \delta_3$. The chara-segments for Mithuna and Karkataka i.e. when Mithuna and Karkataka are rising are to be proved to be equal, here 56 asus. Their expressions are $\tan \phi \tan \delta_2 -$ $\tan \phi \tan \delta_1$ and $\tan \phi \tan \delta_3 - \tan \phi \tan \delta_2$ i.e. $\tan \phi$ $(\tan \delta_2 - \tan \delta_1)$ and $\tan \phi (\tan \delta_3 - \tan \delta_2)$. Since $\delta_1 = \delta_3$ we perceive that they are equal but of opposite signs. So Bhāskara says rightly "अपचीयमानत्वात् धनम्" i.e. because of negative sign, the chara-segment of Karkataka while being subtracted will be rendered positive'. Hence the rising times of Karkataka, Simha and Kanya will be respectively 1937+56, 1793+134, 1670+168 asus or 1993, 1927, 1838 asus or 332, 321, 306 Vinadis or 5-32, 5-21 and 5-6 nadis. Thus, in as much as the rising times of Meṣa to Kanyā are 1670—168, 1793—134, 1937—56, 1937+56, 1793+134, 1670+168 their total is 30 nadis as should be expected because the equator bisects the ecliptic between r and $\simeq$ and the equatorial interval between r and $\simeq$ is 30 nadis. It will be noted that while the equatorial rising times of Meṣa to Kanyā are symmetrical as 1670, 1793, 1937, 1937, 1793, 1670, their rising times at any other place are not like that but

constitute a different kind of symmetry as 1670−168,
1793−134, 1937−55, 1937+55, 1793+134, 1670+168.

We have now to comment upon the statement
"तुलादितोऽसी च विलोमसंस्थाः " which means that the rising
times from Tulā to Mīna are in the reverse order i.e. the
rising time of Tulā equals that of Kanyā ; that of Vris-
chika equals that of Simha and so on the rising time of
Mīna equalling that of Meṣa. Thus the rising times of
Tulā to Mīna being in the reverse order are 1670+168,
1793+134, 1937+55, 1937−55, 1793−134, 1670−168
for the aforesaid locality. Why it should be so can be
easily seen from the fact that Kanyā and Tulā are sym-
metric with respect to the line $r≏$ which bisects the
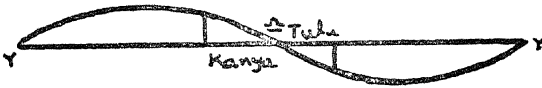ecliptic (Ref. fig. 27). Or again we can see this in another

Fig. 27

way ; the chara-segments are successively ($\tan \phi \tan \delta_1 -$
$\tan \phi \tan 0$), ($\tan \phi \tan \delta_2 - \tan \phi \tan \delta_1$), ($\tan \phi \tan \omega -$
$\tan \phi \tan \delta_2$), ($\tan \phi \tan \delta_2 - \tan \phi \tan \omega$), $\tan \phi \tan \delta_1 -$
$\tan \phi \tan \delta_2$ ($\tan \phi \tan 0 - \tan \phi \tan \delta_1$), ($\tan \phi \tan \delta_1 -$
$\tan \phi \tan 0$), ($\tan \phi \tan \delta_2 - \tan \phi \tan \delta_1$), ($\tan \phi \tan \omega -$
$\tan \phi \tan \delta_2$), ($\tan \phi \tan \delta_2 - \tan \phi \tan \omega$) ($\tan \phi \tan \delta_1 -$
$\tan \phi \tan \delta_2$), ($\tan \phi \tan 0 - \tan \phi \tan \delta_1$) where $\delta_1 =$
declination of 30°, and δ_2 that of 60°. These are there
as found before 56, 134, 168, −168, −134, −56 upto Kanyā.
But the remaining, though apparently are 56, 134, 168,
−168, −134, −56 must be taken with a reverse sign because
the rising point of the ecliptic will be to the south of the
east point send δ will be negative from 180° to 360° longi-
tude. Hence the chara Segments are 56, 134, 168, −168,
−134, −56, −56, −134, −168, 168, 134, 56 so that from
Tulā onwards they are in the reverse order as Bhāskara

has stated. Bhāskara's statement yet has another mean-
ing. The ecliptic from 180° to 360 being in the reverse
as can be seen in fig. 27, the time by which a particular
Rāśi rises, is equal to the time by which its seventh Rāśi
or diametrically opposite Rāśi sets. It is important to
note that the rising time of a particular Rāśi is not equal
to its setting time as can be gauged from the rising times,
for, then the rising times of Meṣa and Tulā must be equal
which is not the case. The word विलोमसंस्थाः has this
meaning as well, rising and setting being reverse directions.

Incidentally there is what is called a
handed down i.e. a traditional statement which men-
tions " मीनमेषौ चतुर्नाड्य,: सार्धं चत्वारिंगोघटौ etc." This
clearly pertains to the local rising times of the Sāyana
Rāśis at a place of latitude 17°–45' i.e. in between
Rajamundry and Vizianagaram, as in this latitude the
chara Segments in asus will be 230, 184, 73 which give
rise to such rising times.

We next find what are called Nirayana Swōdayas i.e.
the rising times for the locality of the Nirayana Rāśis i.e.
the Rāśis from the zero-point of the Hindu zodiac. We
shall obtain these magnitudes for Rajahmundry whose
latitude is $17°–2'$. We shall however take it as $17°$; also
we shall take the Ayanāmśas to be $21°$ at present following
Varāhamihira. This is one of the contexts where the
knowledge of Ayanāmśas is essential. The method is the
same as followed before; only, we have to find the decli-
nations, right ascensions, charas and therefrom the rising
times of arcs of magnitude $21°$, $51°$, $81°$......$351°$. Sub-
tracting the preceding rising times from the following we
have successively the required rising times. We shall
give them under the following table.

Longitude	21°	51°	81	111	141	171	201
Declination	8-13	18-13	23-12	21-51	14-32	3-37	8-13
Right ascension	19-24	48-14	80-12	112-43	143-24	171-44	199-21
Chara (ascentional difference)	2-30	5-41	7-32	7-3	4-33	1-6	2-30
Rising time in Vınādıs	169	429	727	1057	1389	1706	2019
Longitude	231	261	291	321	351	21°	
Declination	18-3	23-12	21-51	14-32	3-37		
Right ascension	228-33	260-12	292-42	325-24	351-44		
Chara	5-41	7-3	4-43	4-43?	1-6		
Rising time in Vınādıs	2342	2677	2998	3279	3528		

In modern terms, the rising times of Rasis could be found by solving the spherical triangle rEA (fig. 28) using the Inner side inner angle formula namely
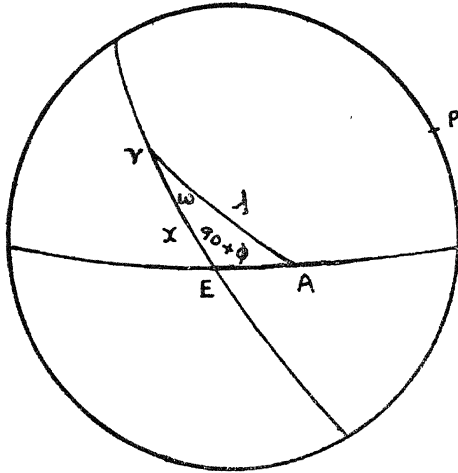

Fig. 28

$$\cos x \times \cos \omega = \sin x \cot \lambda + \sin \omega \tan$$
$$\sin x \cot \lambda - \cos x \cos \omega = - \sin \omega \tan$$

putting $\sin x = t$, this reduces to

$$t \cot \lambda - \sqrt{1 - t^2} \qquad = - \sin \omega \tan$$

or $(t \cot \lambda + \sin \omega \tan \phi)^2 = (1 - t^2) \cos$

$$\therefore \quad t^2 (\cos^2 \omega + \cot^2 \lambda) + \quad \times \sin \omega \tan \phi \cot \lambda - \cos^2 \omega = 0$$

$$t =$$

$$-\sin \omega \tan \, \cos \lambda \pm \qquad \cot$$
$$\cos^2$$

Putting successively $\lambda = 30°, 60° \cdots 360°$ and ignoring the negative sign of the radical we have the sines of rising times of arcs of the ecliptic of $30°$, $60°$ etc. Subtracting

26

the preceding from the succeeding, we have successively
the rising times. Thus the rising time of Mesha is

$$\frac{\sqrt 3 \sin^2 \omega \tan^2 \phi + \cos^2 \omega (\cos^2 \omega + 3 - \sqrt 3 \sin \omega \tan \phi}{3 +} \quad \text{II}$$

That of Meṣa and Vriṣabha put together, the rising
time is

$$\frac{\sqrt{\sin^2 \omega \tan^2 \phi + 3 \cos^2 \omega (\cos^2 \omega + \tfrac{1}{3})} - \sin \omega \tan \phi}{\sqrt 3 (\cos^2 \omega + \tfrac{1}{3})} \quad \text{III}$$

The combined rising time of the 3 Rasis Meṣa, Vriṣa-
bha and Mithuna is from I using tables $\cos^{-1} (\tan \omega \tan \phi)$
$= 5046$ asus which exactly accords with what we have
found previously namely $1505 + 1659 + 1882 = 5046$. Also
putting in I $\lambda = 180°$, we have $\sin x = 0$ or $x = 180° =$
10800 asus; subtracting 5046, we have the combined times
of rising of Karkataka, Simha ahd Kavya to be 5754 asus
as we have had. Putting again $\lambda = 270$, we have $\cos^{°-1}$
$(\tan \omega \tan \phi) = 2\pi - 5046$ which signifies that the sum
of the rising times of the last three rasis is the same as
that of the first three which again means that the sum of
the rising times of Tula to Dhanus is equal to the sum of
the rising times of Karkataka, Simha and Kanya establish-
ing Bhaskara's statement " चिर

Verse 60. Computations of Lagna, Udayāntara and
the like from the rising times of big arcs of the ecliptic
like Rasis will be approximate, whereas one desirous of
greater approximation has to find the same from the
rising times of smaller arcs likes Horas and Dṛkkāṇas, so
as to be more correct.

Comm. Rasis divided into halves are called horas
and if divided into one-third parts are called
Dṛkkāṇas. The meaning of the verse is that if after
having found the rising time of a particular Rāśi say
Meṣa, we say that one-third of that rising time is that of
one-third of that Rāśi, we will be making only an appro-

ximate statement just like saying that 'Since 12 Rasis rise in the course of a sidereal day, so each rasi rises in 1800 asus' which is far from truth being based on a crude rule of three. So, Bhāskara says, Acharyas like Aryabhata insisted on finding the rising times of Dṛkkāṇas, in as much, as while computing the Lagna or the point of intersection of the Ecliptic with the horizon, we will be nearer the truth by using the rising times of smaller arcs like Dṛkkāṇas than broadly using those of Rasis.

Verse 61. Bhujāntara correction.

The equation of centre of the Sun multiplied by the equatorial rising time of the Rāsi occupied by the Sun, and divided by 1800, and then again multiplied by the true daily motion of a planet and divided by the number of asus in a day, is the correction in the planet positive or negative according as the equation of centre of the Sun is positive or negative.

Comm. This Bhujāntara correction arising out of the Sun's equation of centre is prescribed even for the Sun, as well as for the other planets. The planets are originally computed for the rising time of the Mean Sun, whereas we are interested to know the positions at the True Sunrise. The position of the Sun also is originally computed for his mean rise and is therefore to be rectified to get his position at his true rise. So even the Sun is not exempt from the correction. It will be noted here that as the equation of centre pertains to the eccentricity of the Sun's orbit, this correction of Bhujāntara is a correction for the so-called modern 'Equation of time due to eccentrity'. In other words, the Sun's equation of centre converted into time is exactly what is called the Equation of time due to eccentricity.

The formula prescribed for the correction is as follows. Suppose the Sun is in a particular Rāsi which rises at the equator in x asus. Let the equation of centre of the Sun be

E minutes of arc. Then the equatorial rising time of E is $\frac{E \times x}{1800}$ asus because there are 1800' in a Rasi. We have to provide a correction in the planetary position including that of the Sun for this time. If the planet goes Y' during 21659 asus of a day, what arc is covered by the planet or the Sun in $\frac{E \times x}{1800}$ asus? The result is $\frac{E \times x}{1800} \times \frac{y}{216}$ minutes of arc. This correction is positive if the equation of centre of the Sun is positive, for, we want the planetary position for a latter time than Mean Sunrise, since the positive equation of centre advances the True Sun over the Mean. This correction will be appreciable only in the case of the Moon having a rapid motion.

Verses 62, 63. The correction known as Udayāntara.

The difference in minutes of arc in the longitude of the Sāyana mean Sun and the asus in his Right ascension, multiplied by the daily motion of the planet and divided by 21659 is the result to be added to or subtracted from the planet's longitude according as the asus in the Sun's Right ascension are greater or less than the minutes of arc of his longitude. This is what is called Udayāntara correction in the planetary position.

Comm. (1) We have seen that the Bhujāntara is a correction in the mean planetary position due to the Equation of time in Eccentricity.

(2) This Udayāntara is a correction in the same due to the Equation of time in obliquity.

(3) Some have misconstrued that this Udayāntara correction is the Equation of time in the obliquity itself, whereas it is a correction to be effected in the planetary position due to the Equation of time due to obliquity.

(4) The maximum equation of centre in the Sun has a magnitude $= 13\frac{2°}{3} \times \frac{1}{2\pi} = \frac{41}{3} \times \frac{7}{44} =$

Converting this into time at the rate of 15° per hour (since in one hour diurnal rotation of the earth is equal to 15°) we have $\frac{13}{6} \times 4$ minutes $= \frac{26}{3} = 8'-40''$. The maximum equation of centre according to modern astronomy is 2e expressed in radiaus where $e = \frac{1}{60}$ (.0167339) =

radiaus $= 2 \times \frac{1}{60} \times \frac{180 \times 7}{22}$ degrees $= \frac{21°}{11} = 1°-54'$ -

As such the max. Equation of time due to eccentricity is $\frac{21}{11} \times 4' = \frac{84}{11} = 7'-38''$. The small difference in the two values arises out of the difference in the max. equations of centre. Any way it is clear that, in as much as the Bhujāntara correction is necessitated on account of the equation of centre in the Sun, to obtain the planetary positions computed for the mean Sunrise at the time of True Sunrise, the Bhujāntara correction is a correction in *the planetary position*, on account of the equation of time due to eccentricity. The max. correction to be effected in even the quick moving Moon amounts to $\frac{8 \times 790}{24 \times 60} = \frac{79}{18} =$

(5) We have said in the translation of the verse. 'The asus in the Right ascention of the mean Sun', where what exactly is stated by Bhāskara is, the time of rising of the small arc covered by the Sun in the particular Sāyana Rasi in which the Mean Sun is, (मुक्तासव:) together with the rising times of the previous Sāyana Rasis covered by the Sun. The meaning is therefore the rising time of an arc of the ecliptic equal to the Sāyana longitude of the ecliptic which is measured by his Right ascension at the

rate of 15° per hour or 6° per nadi or 10 Vinādis per degree or 60 asus per 60 minutes of arc or as many asus as there are minutes of arc in the right ascension of the mean Sun. So, what is stated by Bhāskara is the difference of the minutes of arc in the mean longitude of the Sun and the minutes of arc in his right ascension ie. $(l-\alpha)$ expressed in minutes. We know that the total equation of time arising out of unequal motion in the true longitude of the Sun ie. ⊙ in comparision with the equal motion in his right ascension ie. α is measured by $\odot-\alpha$ which may be expressed as $(\odot-l)+(l-\alpha)$ where l is his mean longitude, $\odot-l$ = Equation of centre and so the time expressed by $\odot-l$ is the equation of time due to obliquity. The difference $l-\alpha$ arises out of the obliquity of the ecliptic and so the time expressed by $l-\alpha$ is the equation of time due to obliquity. We have the modern formula

$$\cos \omega = \frac{\tan \alpha}{\tan l} \text{ so that } \frac{1-\cos \omega}{1+\cos \omega} = \frac{\tan \alpha - \tan l}{\tan \alpha + \tan l} = \frac{\sin \alpha - l}{\sin \alpha + l}$$

$\therefore$ Sin $(\alpha - l) = \tan^2 \omega/2 \sin (\alpha + l)$. As α is very nearly equal to l, we could write, when expressed in radians $\alpha - l = \tan^2 \omega/2 \sin 2l$. Thus the maximum difference between α and l arises when $2l = 90°$ ie. $l = 45$ degrees ie. at the mid-point of the first quadrant; the minimum difference is when $2l = 270$ ie. $l = 135$ ie. at the middle point of the 2nd quadrant. Also the numerical magnitudes of the max. as well as the minimum value are each $\tan^2 \omega/2$ ie. they are equal. Since this is expressed in radians, converting into time the numerical value of the max. and minimum equation of time due to obliquity is 9.87′. Thus we can write $l-\alpha = 9.87′ \sin 2l$. Again where $l = 225$, $2l = 450$ so that $l-\alpha$ will have a positive max.; and again when $l = 315$, $2l = 630$ so that $l-\alpha$ will have a negative maximum value. Also when l has values 0, 90, 180, 270 it is zero. Thus the equation of time due to obliquity is zero at r, ie. the vernal equinox; +ve in the first quadrant

increasing from zero to 45 degrees and then decreasing from 45° to 90°; assuming the value zero at 90°, then negative in the second quadrant negatively increasing from zero to a maximum as l increases from 90° to 135°, and then negatively decreasing from the maximum value to zero at the end of the second quadrant; again behaving in the 3rd quadrant as in the first and in the fourth as in the second.

(6) It was stated by Mr. Mazumdar in his introduction to the Siddhānta S'ekhara of S'ripati published by the Calcutta University as well as by pandit Babuaji Misra, the editor thereof that this Udayāntara correction was first mentioned by Sripati, and it meant equation of time due to obliquity. In fact S'ripati states (Verse I ch. elevan) '' अन्त्यभ्रमेण गुणिता रविवाहुजीवाऽमीष्टभ्रमेण विह्लता, फलकार्मुकेण, वाहो: कलासु रहितास्ववशेषकं ते, यातासवो युगयुजो: पदयो: धनर्णम् ''

l expressed in minutes $- H \sin^{-1} \dfrac{H \sin \odot \ H \cos \omega}{H \cos \delta}$ expressed in asus $=$ what are called elapsed asus and are $+$ve, $+$ve, $+$ve and $-$ve in the successive quadrants. We saw before that $H \sin \alpha = \dfrac{H \sin \odot \ H \cos \omega}{H \cos \delta}$ so that S'ripati meant $l - \alpha$, the former expressed in minutes of arc and the latter in asus, or what is the same $(l-\alpha)$ both expressed in minutes or asus. These give the gain of l over α. Immediately after this verse S'ripati goes to a different topic, and never mentions (as understood from the printed text) any further details as to what is to be done with these Yātāsus. Since Bhāskara says explicitly that for these Yātāsus, the planets are to be corrected, we may surmise that there should have been in S'ripati's text also another verse detailing the usage of those asus.

(7) We shall now attend to what Bhāskara gives by way of explanation of the verses in question, The Ahar-